Investor Beliefs about Transformative Innovations

Johannes Binswanger[*]   Anja Garbely[†]   Manuel Oechslin[‡]

May 9, 2022

Abstract

Periodically, economies produce potentially transformative innovations, but the required investments are fraught with large uncertainties. Initially, objective information for predicting their success is scarce. How then do investors form subjective beliefs about prospective returns? We explore the role of competing financial intermediaries that channel funds either to firms in the innovation sector or to firms potentially displaced by the innovation. In our model, investors' subjective beliefs emerge from a competitive campaign game, or *belief contest*, between financial intermediaries. A regulator who does not possess superior knowledge may want to tilt beliefs away from "exuberance" or "pessimism" towards "impartiality".

[*]*Corresponding author.* University of St. Gallen, Department of Economics, Bodanstrasse 8, CH-9000 St. Gallen, Switzerland; johannes.binswanger@unisg.ch.

[†]University of Lucerne, Department of Economics, Switzerland; anja.garbely@unilu.ch.

[‡]University of Lucerne, Department of Economics, Switzerland; manuel.oechslin@unilu.ch.

1 Introduction

Market economies regularly produce innovations in the form of novel technologies or business models that are potentially transformative. Prominent examples include the electric motor, genetic engineering, the Internet, digitalization, artificial intelligence, and business models such as Internet-based retailing and securitization of loans. Naturally, at the time a potentially transformative innovation (PTI) emerges, predicting its success is hard; even experts' assessments are often only marginally better than random guessing, and sometimes worse (Albright, 2002; Goldfarb and Kirsch, 2019; Savage et al., 2021). An important cause of this low predictability is the interaction of multiple sources of uncertainty, relating to technological, competitive, business model, regulatory, and demand factors of uncertainty (David, 1990; Goldfarb and Kirsch, 2019).

A prior belief about the prospects of a PTI that qualifies as "objective" should take into account this uncertainty in the form of a wide range of possible outcomes that may occur with non-negligible probabilities. Nevertheless, substantial evidence shows that the relevant actors often hold beliefs with a large probability mass concentrated on a narrow confidence band (Tversky and Kahneman, 1974; McKenzie et al., 2008; Hirshleifer et al., 2012; Kaustia and Perttula, 2012; Ben-David et al., 2013; Shiller, 2017, 2019). Such beliefs cannot easily be seen as objective but represent subjective prior beliefs, possibly subject to cognitive bias. However, it is noteworthy that the standard notion of rationality as used in economics does not constrain subjective prior beliefs, but only requires that updating of beliefs is consistent with Bayes' rule (see Gilboa, 2009; Gilboa et al., 2014).

In this paper, we consider subjective beliefs of investors potentially interested in PTIs. In reality, potential investors do not acquire their subjective priors in a vacuum. Rather, these priors are shaped in an environment where concerned parties are the most likely actors to initiate "talk" about PTIs (Shiller, 2002). These parties include the firms active in the development of a PTI, but also the financial intermediaries (FIs; e.g. investment banks) that channel funding from investors to the PTI firms. Further important parties are businesses relying on an established technology that potentially gets disrupted by the PTI, as well as their respective financial intermediaries. Crucially for our analysis, this talk may have very limited predictive power for the eventual business success of a PTI.

In our analysis, we focus on the role of FIs in shaping investors' subjective beliefs. We concentrate on FIs rather than the firms that they fund since FIs are specialized in channeling funding from investors to firms. Hence, it belongs to their realm to market investment oppor-

tunities and—if possible—influence investors' beliefs in a way that aligns with this business. We understand these influence activities as a competitive campaign game between FIs specializing in "new economy" PTIs and FIs specializing in funding established—and potentially threatened—businesses ("old economy").[1] We analyze a situation where investors are exposed to the campaigns of both FI types and their belief is determined by the "lead" of one campaign over the other. Formally, we capture this by a contest game or—as we call it—a *belief contest*.

In light of the mentioned literature, from an objective point of view, any early talk communicated by the FIs is likely to be approximately uninformative. So why should the FIs ever stand a chance to shape investors' *prior* belief to their own advantage? As a starting point, empirical evidence shows that investors do not always properly discount the role of FIs' self-interests and follow advice that is not (fully) to their own benefit (Shiller, 2002; Mullainathan et al., 2012; Bergstresser et al., 2008; Hoechle et al., 2018; Egan, 2019). A plausible reason for this is that investors may consider FIs as experts. FIs are among the first in line to observe what happens on the ground in the development of a PTI. Cognitive biases—"System 1" in the language of Kahneman (2003, 2011)— may lead investors to erroneously enhance this expertise to a capability of producing accurate assessments of the overall future business prospects of a PTI. They may thereby ignore that current observations about technical feasibility etc. may be of little predictive value for overall future business prospects (McKenzie et al., 2008; Goldfarb and Kirsch, 2019). A second reason investors may listen to FIs is that their campaigns feature aptly crafted narratives that strike a chord with investors and successfully sideline their critical-thinking "System 2" (Kahneman, 2003, 2011; Martens et al., 2007; Shiller, 2002, 2017, 2019). Finally, the fact that there may be a fierce contest between the two FIs may foster the illusion that whatever is left as a "lead" of one campaign over the other has withstood a high degree of scrutiny—comparable to a court case or an electoral contest.

Investors' subjective belief is crucial for their a priori willingness to invest in a PTI, as well as their propensity to liquidate investments in case of an unfavorable objectively informative signal that may appear at a later stage. They are therefore essential for innovation-based growth and the efficient allocation of capital in an economy. Hence, a belief contest poses important questions for financial regulators, not least regarding financial consumer protection (FCP).[2] In the paper, we consider the role of a regulator who does not have any superior objective information about the state of the world. However, the regulator is aware that

---

[1] For our analysis, it need not be that an FI as a whole specializes in the "new" or "old" economy, but that they consist of departments that specialize in one or the other segment and are in competition to each other, e.g. concerning which department head gets promoted to a top position in the headquarter company.

[2] FCP regulations include rules concerning, amongst others, deceptive advertising and disclosure of product info. They also specify the powers of the regulator to impose sanctions on offenders (e.g., Gaganis et al., 2020).

predictability of the success of PTIs is low, and that investors form their belief according to the described belief contest. The regulator has a possibility of tilting the belief contest in favor or against the PTI.

In our model, a generally favorable prior belief towards new-economy investments is a mixed blessing from the perspective of efficient capital allocation. On the one hand, attracting investors to PTIs requires that investors hold a relatively firm belief about their eventual success. On the other hand, a firm belief can blind investors to negative objectively informative signals that appear at a later stage, thereby raising the risk of severe capital misallocation.[3] We argue that it is precisely this trade-off that can shape a financial regulator's objective in the presence of low objective predictability of success of a PTI. In particular, a regulator may want to maximize the chance that investors adopt an "impartial" prior belief. Under an impartial prior, PTIs are explored, but investors are attentive to objectively informative signals and ready to pull the emergency brake in time if necessary. The model shows that the regulatory stringency necessary to achieve this objective depends on three factors: the ratio of fees the FIs charge to the firms that they help financing; the quality of a later-stage public and objectively informative signal; and the potential productivity gain from the PTI. So, our analysis suggests that a regulator may want to gather and evaluate data on, among other things, the relative profitability of investment banks' (IPO) activities and the degree to which an innovation lends itself to testing and small-scale experimentation.[4]

This paper connects various strands of literature in economics and finance. By arguing that in many situations concerning PTIs there is no objective way to establish probabilities for contingencies, we follow a recent literature on welfare criteria and regulation under uncertainty (e.g., Brunnermeier et al., 2014; Gilboa et al., 2014; Buss et al., 2016). Just as in that literature, we assume that individuals maximize utility under a subjective (prior) belief and that the social planner or regulator does not have any superior information. We further pursue the idea that under uncertainty investors' prior beliefs are liable to the influence of narratives (e.g., Tversky and Kahneman, 1974; Martens et al., 2007; Shiller, 2002, 2017). The idea that self-interested actors try to influence an "audience" in order to win benefits is also present in (Binswanger and Oechslin, 2021). In this complementary paper, we consider a more general setting that allows for heterogeneity in the "audience". However, as the model is static, there is no stage with an objectively informative signal. Furthermore, we do not consider regulation.

---

[3]These are also exactly the trade-offs associated with overconfidence among a company's top management and among venture capitalists; see Hirshleifer et al. (2012) and Graves and Ringuest (2018) for a discussion.

[4]Results from testing and small-scale experimentation are only weak signals if a meaningful test requires a rare event (in the case of securitization: a housing bust) or if network effects are important (e.g., social media platforms).

The idea of a narrative entrepreneur, reminiscent of our FIs in the belief contest, appears in Bénabou et al. (2020).[5] Furthermore, our analysis connects to previous theoretical work on the role of information in financial intermediation (e.g., Bolton et al., 2007; Stoughton et al., 2011). But in contrast to existing papers, and consistent with our focus on PTIs, we do not assume that ex ante FIs hold an objective informational advantage over their clients; we consider intermediation and investment in a setting where initially the absence of objectively informative evidence means a lack of discipline on narratives and priors, while objective information gradually emerges over time.

Our analysis also relates to persuasion games, which come in several varieties. First, in Bayesian persuasion games (Kamenica and Gentzkow, 2011; Kamenica, 2019), a sender commits to disseminating informative signals as a function of the state of the world with the goal to influence the beliefs and hence also the actions of a receiver. This may be possible even though the receiver is fully rational and knows that the sender designs strategically the signal-generating process; and realized signals are revealed truthfully regardless of the realization. Second, in signaling games (Spence, 1973; Cho and Kreps, 1987), the sender is informed about the state of the world and chooses the signals given his information, in order to influence the receiver's actions towards the one that is most favorable to the sender. In comparison to Bayesian persuasion games, for every state, any signal can be chosen and hence the receiver eventually discounts the information in the signal as it is chosen strategically by the sender. This is so even if different signals have different costs for the sender depending on the state. Finally, in cheap talk games (Crawford and Sobel, 1982), all signals are costless but there exist equilibria where different signals induce different beliefs and hence influence the action of the receiver in a way that is favorable to the sender. In all these games, it is implicitly assumed that the distribution of the states of the world is common knowledge and is agreed upon by all agents, i.e. sender and persuader. Moreover, the sender has objective information about the state of the world. Both assumptions do not hold in our case, and investors mistake cheap talk for a signal. In all mentioned games, the analysis of our setting would be trivial: investors in the role of receivers would hold a prior according to which they know that FIs do not have access to any objective information; and hence they would not react to any signal.

The remainder of this paper is organized as follows. The next section briefly discusses the case of online retailing associated with the dot-com bubble. Sections 3 and 4, respectively, introduce and solve the formal model. In Section 5, the focus lies on regulation. Section 6

---

[5]Mokyr (2013, 2016) distinguishes between institutions and culture and notes that beliefs are part of the latter. In his works, actors who try to influence other individuals' beliefs are called "cultural entrepreneurs".

concludes.

2 The Case of Online Retailing ("Dot-Com")

To provide a concrete example of the kind of constellation our model is meant to capture, we look back to the 1990s. In that period, fast progress in computer and information technology caused a sharp fall in the cost of storing and sending information. As a result, dot-com companies sprung up like mushrooms, not least in the retail sector. Traditional bricks-and-mortar retailers (old economy) were suddenly confronted with competition from online retailers (new-economy PTI). The competition extended to the financial markets as soon as the new online retailers started to seek large amounts of capital by going public. These undertakings were carried out by investment banks such as Morgan Stanley Dean Witter or Merrill Lynch (new-economy FIs) that were experienced in technology IPOs and were able to charge large fees for the corresponding services (e.g. Kindleberger and Aliber, 2015). At the same time, although lack of experience meant that the eventual success of many online businesses was highly uncertain from an objective point of view, the investment banks spread positive narratives, for instance via bullish research reports by their dot-com stock analysts (e.g. Cassidy, 2003). Some analysts, such as Morgan Stanley's Mary Meeker even gained celebrity status and were instrumental in delivering the respective narratives effectively.

The narratives from both sides were based on analogies with a strong intuitive appeal. The analogy at the heart of many new-economy narratives was that the Internet was like the "land grab" (Westward expansion in the US), where time was of the essence and the first-mover advantage substantial. According to an article published in *The Times* (Aug 10, 1999), Sascha Mornell, then vice president of the online start-up register.com, described it as follows:

> "It's like the landgrab all over again. Cyberspace is the new frontier. The Internet is virgin territory and as far as the eye can see there are millions of unclaimed acres out there."

An immediate corollary of the land-grab narrative was that traditional valuation measures, such as price-to-earnings ratios, would be outdated and not relevant for dot-com start-ups. These businesses would have to be evaluated based on their potential profits and not their current profits. This reasoning was echoed in an article published in the *Wall Street* (May 19, 1999) approximately one year before the peak of the dot-com bubble:

> "One of the sacred tenets of business—you have to make money—suddenly looks almost like a quaint artifact of an outdated era. Many investors care less about a company's being

in the black than about its ability to win turf in what has become a giant land grab."

The role of the old-economy FI was assumed by managers of value-oriented mutual funds, among others. These mutual funds stuck to clear investment principles that stopped them from investing in dot-com start-ups early on. An article published in the Wall Street Journal (Mar 6, 2000) suggests that fast rising dot-com stocks put value-oriented mutual funds at the risk of being perceived as under-performing (and thus of losing investors and fee income). So mutual-funds managers had an interest in spreading pessimistic narratives, among them ones that offered as analogies past great bubble episodes such as the "railway mania". This is echoed in an article published in The Economist (Sep 23, 2000):

> "There are many similarities between the Internet today and Britain's railway mania in the 1840s. Would-be rail millionaires raised vast sums of money on the stockmarket to finance proposed lines. Most railway companies never paid a penny to shareholders, and many went bust [. . .]."

Essentially, managers of value-oriented mutual funds contended that times were not different, after all, and Internet start-ups had to be evaluated in the same way as bricks-and-mortar companies, using traditional valuation metrics such as the price-to-earnings ratio. Their main message was that most of the stocks were overpriced when applying traditional valuation metrics and thus would collapse soon, just as in earlier such episodes.

Hence, both sides used narratives (in the form of historical analogies) to influence the beliefs of investors. For quite some time, the land-grab narrative proved to be more powerful than the railway-mania narrative. Online retailers could raise large amounts of capital well into the year 2000. Warning signs, including repeated big losses incurred by many of them, were played down, not least with reference to the supposedly new valuation metrics. But later that year, reality started to sink in, and investors withdrew in droves. For many online retailers, this was the end. According to an article published on IDEAS.TED.COM (Dec 4, 2018), by 2002, investors had lost about $5trn in the stock market.

3 The Model

3.1 The Real Sector

Our model encompasses a segment of the overall economy consisting of a real and a financial sector. The real sector represents a particular industry, or a collection of industries that use

a common technology or business model. The real sector provides investment opportunities in terms of productive physical assets. Specifically, there is a mass-one continuum of PTI investment opportunities ("new economy"), indexed by $i \in [0, 1]$, which can be thought of as individual firms. The rate of return (net of depreciation) offered by opportunity i depends on the fundamental state of the economy, $F \in \{H, L\}$:

$$r_n(F) = \begin{cases} \alpha^h > 0 & : & F = H \\ \alpha^l < 0 & : & F = L \end{cases}, \tag{1}$$

where $\alpha^l = -\alpha^h > -1$. This symmetry assumption simplifies the exposition but is not critical for our results. At the stage where investors make their investment decision, objective information on the probabilities with which the possible values of F materialize is completely absent.[6] As specified in the next subsection, investors adopt a subjective prior belief p about the probability of $F = H$ that emerges as the outcome of a belief contest between FIs. This prior allows investors to calculate a subjectively expected rate of return.

At a later stage, after investors have chosen their initial investment opportunities, a publicly observable signal $S \in \{H, L\}$ appears that contains objective information about F. Signal quality, a known parameter, is given by

$$\sigma \equiv \Pr[S = F] > 1/2 \ \ \text{for} \ \ F, S \in \{H, L\}. \tag{2}$$

Obviously, a larger value of σ means a more precise signal: as σ increases from $1/2$ to 1, the signal's quality monotonically improves from completely uninformative to perfect. A condition for a high signal quality may be that the new technology lends itself to early informative testing, for example by experimental introduction in a small market segment—which may be infeasible if the new technology's benefits depend, e.g., on strong network effects.

After having observed signal S, investors use Bayes' rule to update their prior belief regarding the probability of $F = H$.[7] The resulting posterior is given by

$$q(S, p) \equiv \Pr_p[F = H \,|\, S], \tag{3}$$

---

[6]The empirical literature that we refer to in the introduction uses the width of confidence bands in the return domain as measures of overconfidence. For tractability, our model considers only two return outcomes and confidence will relate to the probabilities with which these outcomes materialize (see next Subsection below).

[7]It is a valid question whether Bayesian updating is an adequate model for agents whose beliefs are influenced by campaigns from self-interested parties (see next Subsection below). However, Bayesian updating provides a natural benchmark. Inferring how our results change as a consequence of biased updating, e.g. in the form of stickiness towards the prior, is straightforward.

where we use subscript "p" to indicate that the posterior is calculated under prior p. Equipped with the updated belief, PTI investors can reconsider their decision. More specifically, any PTI investment $i \in [0,1]$ can be liquidated if the group of investors involved chooses to do so. As we show below, there will always be unanimous consent in liquidation matters among the involved investors. Liquidation entails that physical assets are not utilized for production with the innovative technology. However, unused physical assets do not depreciate and can be employed in a future productive process.[8] It follows from these assumptions that liquidation results in a rate of return of zero. So, on the whole, the rate of return on PTI investments, R_n, takes one of three different values:

$$R_n \in \{\alpha^h, \alpha^l, 0\}. \tag{4}$$

Besides the PTI investment opportunity, the economy also features a mass-one of investment opportunities relying on established technologies or business models ("old economy"), indexed by $j \in [0,1]$. We assume that the corresponding rate of return is riskless:

$$r_o = \beta \in (0, a^h), \tag{5}$$

An obvious real-world equivalent of what we call old-economy investments are corporate bonds issued by long-established businesses. While these are not risk-free in reality, their risk is clearly significantly lower than for PTIs, and little insight is lost by making this simplifying assumption.[9]

The ratio $\alpha^h/\beta > 1$ is the productivity increase the PTI brings about in case of a favorable fundamental state (i.e., $F = H$); it represents the "promise" of the PTI. In what follows, our analysis will concentrate on an innovation with a sufficiently high promise:

$$\alpha^h/\beta > (\sigma - 1/2)^{-1}. \tag{R1}$$

We are going to clarify the meaning of restriction (R1) in Section 4 below. Essentially, assuming a "low-promise" innovation would amount to defining the problem of interest away.

---

[8] Again, this assumption is for simplicity only and not critical for our results. What we need for the model to work is that any liquidation-specific rate of depreciation is strictly less than the absolute value of α^l.

[9] Appendix A states a set of assumptions regarding technology and depreciation that implies equations (1) and (5).

3.2 Financial Sector and Subjective Beliefs

The financial sector of the economy consists of a large number of investors and two FIs. Each investor offers a given amount of savings, with I denoting the total amount available for investment, while I_n and I_o refer to investment in the new and the old economy, respectively. Without loss of generality, I is normalized to one. The two intermediaries channel savings to the investment opportunities in the real sector. Both are specialized: the new-economy FI tries to broker investment opportunities in the PTI, while the old-economy FI does the same for opportunities based on established technologies.

The FIs charge fees for their services. If PTI firm i receives funding, it owes the new-economy FI a fee of $v^{nf} > 0$; the fee falls due irrespective of investors' decisions on liquidation at a later stage. The corresponding fee charged by the old-economy FI is $v^{of} > 0$. In our model, fees are exogenous. In practice, the fee levels may be inversely related to the intensity of competition in the corresponding segments of the financial sector (e.g., in IPO and bond underwriting, respectively).[10] It is noteworthy that, in reality, an FI may not specialize in either the "new" or the "old" economy overall, but rather may consist of specialized departments. We believe that our model can still adequately capture this situation as long as these departments are in competition to each other, e.g. concerning their budgets, or which department head gets promoted to a top position in the headquarter company.

For simplicity, investors are risk-neutral and thus choose the investment opportunity for which they expect the highest rate of return. Expectations regarding PTI opportunities are based on a common subjective prior belief p about $F = H$. This belief emerges as the outcome of a belief contest, which we model according to a standard contest success function. Denote campaign efforts by the new- and the old-economy FI by $x_{nf} \geq 0$ and $x_{of} \geq 0$, respectively. Let $\zeta \geq 0$ denote a random shock. Then, the prior belief about $F = H$ is determined according to

$$p(x_{nf}, x_{of}) = \frac{\zeta\,(x_{nf})^{\delta}}{\zeta\,(x_{nf})^{\delta} + (x_{of})^{\delta}} \tag{6}$$

whenever either $x_{nf} > 0$ or $x_{of} > 0$; and $p = 1/2$ otherwise. The underlying idea is that campaigns are fought by means of suitable narratives and x_{nf}, x_{of}, capture efforts put into the respective campaign. This includes time invested into a careful crafting of persuasive narratives, the recruitment of supporting celebrities, or the writing of white papers, etc. The random shock ζ captures the role of luck, e.g. related to the timing of a celebrity endorsement

---

[10]The levels of the fees cannot be arbitrarily high. Appendix A establishes upper limits that ensure that it is indeed optimal for a PTI or old-economy firm to seek funding via the corresponding FIs.

etc.[11] Realistically, campaigns of both FIs are subject to luck. However, the functional form of the contest success function implies that, if there were two such shocks, one for each FI, only their ratio would matter for the outcome. Hence, we can understand ζ as representing this ratio; it augments the productivity of campaigning efforts of the new-economy FI relative to those of the old-economy opponent. If $\zeta > 1$ ($\zeta < 1$), the campaigning efforts by the former are relatively (un-)productive. The relative productivity shock, in turn, depends on two factors:

$$\zeta = \tilde{\zeta}/\lambda, \tag{7}$$

where $\tilde{\zeta}$ is a random variable and λ is a parameter. We assume that $\tilde{\zeta}$ follows an exponential distribution with a mean of one. The exponential distribution has a long right tail. So, while the belief contest will more often be tilted in favor of the old-economy FI (median < mean), there is a chance that a positive narrative "goes viral"—and greatly sways the contest in favor of the new-economy FI.

The second factor in ζ, $1/\lambda$, captures (FCP) regulation, with stringency rising in the level of $\lambda > 0$. For a given realization of $\tilde{\zeta}$, a tightening of the regulatory screws lowers the relative productivity of the spending by the new-economy FI. The logic behind this is simple: more stringent rules against deceptive advertising and concealment of potential risks, or harsher punishments for breaking those rules, primarily constrain the new-economy FI who is making the case in favor of the uncertain investment opportunities. For instance, with a view to possible lawsuits in case of strongly negative returns, it may be optimal to use a more cautious exposition in the advertising material if possible punishments are harsher. In Section 5, we consider the parameter λ as a choice variable of the regulator.

It remains to discuss $\delta \in (0, 1)$.[12] This parameter governs the decisiveness (Hirshleifer, 1995) of the contest, i.e., the degree to which differences in campaigning efforts translate into differences in terms of contest outcome. We do not give δ a structural interpretation, but rather consider it a technical parameter (whose function is described in Footnote 12).

The FIs know the parameters of the contest success function (6) as well as the distribution of $\tilde{\zeta}$. They simultaneously choose x_{nf} and x_{of} to maximize their payoff functions

$$V(x_{nf}, x_{of}) = \mathbb{E}\left[\int_0^1 \mathbb{1}_{n,i}\left[p(x_{nf}, x_{of}), \cdot\right] di\right] v^{nf} - x_{nf} \tag{8}$$

---

[11]See Shiller (2019) for an extensive discussion of the role of luck in the effectiveness of narratives.

[12]The restriction $\delta < 1$ is common in applications of the "standard" Tullock contest, where it guarantees the existence and uniqueness of a pure-strategy Nash equilibrium (see Szidarovszky and Okuguchi, 1997). In our case, this restriction simplifies the verification of the existence of an equilibrium, without guaranteeing one.

and

$$W(x_{nf}, x_{of}) = \mathbb{E}\left[\int_0^1 \mathbb{1}_{o,j}\left[p(x_{nf}, x_{of}), \cdot\right] dj\right] v^{of} - x_{of}, \tag{9}$$

for the new-economy and old-economy FI, respectively. $\mathbb{1}_{n,i}[p, \cdot]$ is a binary indicator variable that takes a value of one if PTI opportunity i receives funding and a value of zero otherwise. The indicator $\mathbb{1}_{o,j}[p, \cdot]$ is defined analogously. The notation in payoff functions (8) and (9) highlights that the binary indicators are functions of the campaigning effort, among other factors.

3.3 Timing

The game proceeds in (maximally) five stages:

1. Nature determines the fundamental, yet unobserved, state of the economy, $F \in \{H, L\}$.

2. The intermediaries simultaneously choose the level of campaigning efforts for the belief contest, $x_{nf}, x_{of} \geq 0$; at the same time, nature draws a value for $\tilde{\zeta}$ from $\mathrm{Exp}(1)$.

3. Investors adopt $p(x_{nf}, x_{of}; \zeta)$ as their common subjective prior belief about the probability of $F = H$; they then choose in which part of the real economy to invest.

The game continues if in stage 3 PTI opportunities have got funding:

4. Nature determines the objectively informative but noisy signal $S \in \{H, L\}$ about state F.

5. PTI investors form their posterior belief $q(S, p)$; they then decide on whether or not to liquidate their investments.

The payoffs materialize either after stage 3 or 5. If in stage 5 some PTI investments are continued, the rate of return on those investments, $r_n(F)$, reveals state F.

The above game is one with complete but imperfect information. We now solve for the sub-game-perfect equilibrium of the game by going backwards through the stages.

4 Analysis: Financial Intermediaries and Investors

4.1 Decision on Liquidation

Consider a PTI investment opportunity i that has got funding in stage 3 of the game and focus on the situation of an arbitrary investor who holds a stake in it. In stage 5, after having

Figure 1: The posterior belief as a function of S and p
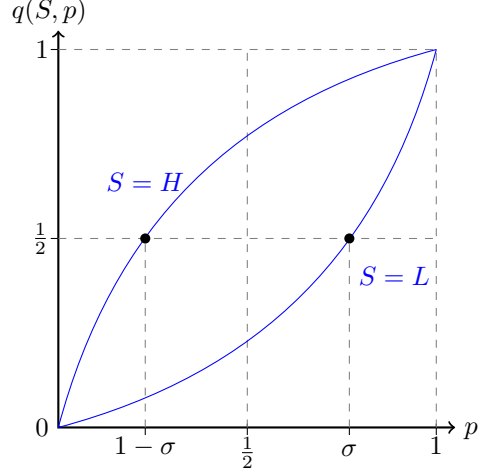
received signal $S \in \{H, L\}$ in stage 4, the investor has to decide whether to favor continuation or liquidation of the investment. The investor's decision depends on $q(S, p)$, the posterior belief about the probability that the PTI will deliver the high rate of return ($F = H$). Because of symmetric returns ($\alpha^l = -\alpha^h$), the expected rate of return associated with continuation is non-negative if and only if $q(S, p) \geq 1/2$. The investor thus prefers continuation if this condition is met, and liquidation otherwise. As the investor's posterior is universally shared, everyone else who has invested in i agrees. Investment i thus continues to exist if $q(S, p) \geq 1/2$ and is liquidated otherwise.

When it comes to the investors' subjective posterior belief, it follows from Bayes' rule that

$$q(S, p) = \begin{cases} \left[1 + \frac{1-p}{p} \frac{1-\sigma}{\sigma}\right]^{-1} & : \quad S = H \\ \left[1 + \frac{1-p}{p} \frac{\sigma}{1-\sigma}\right]^{-1} & : \quad S = L \end{cases}. \tag{10}$$

Since $\sigma > 1/2$, equation (10) implies that $q(H, p) > q(L, p)$ for all $p \in (0, 1)$. Moreover, as illustrated in Figure 1, both posteriors are monotonically increasing functions of the prior, rising from a minimum of 0 (for $p \to 0$) to a maximum of 1 (for $p \to 1$). The properties of $q(S, p)$ imply that the decision on whether or not to liquidate an arbitrary PTI investment i does not necessarily depend on the realization of S. On the one hand, if p is sufficiently small such that $q(H, p) < 1/2$, i always faces liquidation. On the other hand, if p is sufficiently large such that $q(L, p) \geq 1/2$, the investment continues to exist regardless of what the signal says. Only if $q(H, p) \geq 1/2$ and $q(L, p) < 1/2$, the signal is pivotal for i's survival. As none of the

above arguments is specific to investment opportunity i, the analysis equally applies to any PTI investment. We therefore conclude the following:

PROPOSITION 1 *In stage 5 of the game, suppose that there exist PTI investment opportunities that have received funding in stage 3. When it comes to the investors' decisions on whether these investments continue to exist or are liquidated, signal S is pivotal if and only if*

$$1 - \sigma \leq p < \sigma, \tag{11}$$

where $S = H$ implies continuation and $S = L$ implies liquidation. Otherwise, these investments always continue to exist (if $p \geq \sigma$) or are always liquidated (if $p < 1 - \sigma$).

Proof. The proposition follows from the exposition in the text above. To obtain the two thresholds in equation (11), use the corresponding expressions in equation (10). ∎

4.2   Decision on Investment

In stage 3 of the game, all investors calculate $\mathbb{E}_p[R_n]$, the expected rate of return on PTI investments under prior p. Proposition 1 implies that the investors have to distinguish between three different cases. First, if $p < 1 - \sigma$, potential PTI investors anticipate that they would unconditionally liquidate any new-economy investment in stage 5. Therefore,

$$\mathbb{E}_{p \in (0, 1-\sigma)}[R_n] = 0. \tag{12}$$

Second, if $1 - \sigma \leq p < \sigma$, potential PTI investors know that their liquidation decision in stage 5 would be determined by the signal. Accounting for this, we get

$$\mathbb{E}_{p \in [1-\sigma, \sigma)}[R_n] = \alpha^h(p + \sigma - 1), \tag{13}$$

where α^l is eliminated from the equation using $\alpha^l = -\alpha^h$. Third, if $\sigma \leq p$, potential PTI investors anticipate that they would continue their investment regardless of the signal's realization in stage 4. Bearing in mind that $\alpha^l = -\alpha^h$, we get

$$\mathbb{E}_{p \in [\sigma, 1)}[R_n] = \alpha^h(2p - 1). \tag{14}$$

Figure 2 illustrates that $E_p[R_n]$ is a continuous and piecewise linear function of p on $[0, 1]$. In addition, the figure indicates that the function is monotonically increasing, rising from a

Figure 2: The expected rate of return on PTI investments as a function of p
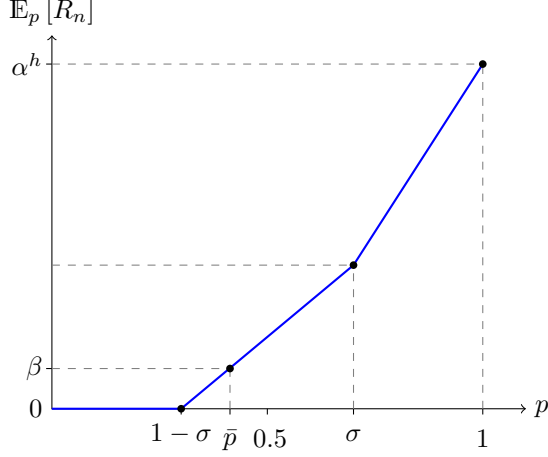
minimum of 0 (for all $p \le 1 - \sigma$) to a maximum of α^h (for $p = 1$). Because $0 < \beta < \alpha^h$, the properties of $\mathbb{E}_p[R_n]$ imply that there exists a critical threshold $\bar{p} \in (0, 1)$, implicitly defined by

$$\mathbb{E}_{\bar{p}}[R_n] = \beta, \tag{15}$$

such that $\mathbb{E}_p[R_n] < \beta$ for all $p < \bar{p}$ and $\mathbb{E}_p[R_n] > \beta$ for all $p > \bar{p}$. By way of illustration, Figure 2 assumes an arbitrary value for β (vertical axis) and indicates the corresponding $\bar{p}$ (horizontal axis). The following conclusion is now immediate:

PROPOSITION 2 *In stage 3 of the game, there exists a critical subjective prior belief $\bar{p} \in (0, 1)$ such that all investors place their investments in old-economy opportunities if $p < \bar{p}$; all investors place their investments in PTIs if $p \ge \bar{p}$.*

Proof. The proposition follows from the existence of a $\bar{p}$ with the properties discussed in the text above, in combination with risk neutrality on the part of the investors. ∎

In Figure 2, the critical subjective prior belief $\bar{p}$ does not exceed the threshold $\sigma > 1/2$. This is not by accident: owing to restriction (R1), it follows that

$$\bar{p}(\alpha^h/\beta, \sigma) = 1 - \sigma + \left(\alpha^h/\beta\right)^{-1} < 1/2. \tag{16}$$

Equation (16) implies that the critical prior is a decreasing function of both α^h/β and σ. The equation also clarifies the meaning of restriction (R1): with respect to the novel technology, "promising" means that a subjective prior of $1/2$ is sufficient to induce investors to try it out.

15

4.3 Decision on Contest Expenditures and Equilibrium Beliefs

From Proposition 2 we know that the new economy either attracts all investors ($I_n = 1$) or no investors at all ($I_n = 0$). As a result, payoff function (8) simplifies to

$$V(x_{nf}, x_{of}) = \Pr\left[p(x_{nf}, x_{of}, \zeta) \geq \bar{p}\right] v^{nf} - x_{nf}. \tag{17}$$

Using contest success function (6), equation (17) can be rewritten as

$$V(x_{nf}, x_{of}) = \left\{ 1 - D\left[\lambda \frac{\bar{p}}{1 - \bar{p}} \left(\frac{x_{of}}{x_{nf}}\right)^{\delta}\right] \right\} v^{nf} - x_{nf}, \tag{18}$$

where $D[\cdot]$ refers to the distribution function of the exponential distribution. Similar reasoning implies that payoff function (9) can be restated in the following way:

$$W(x_{nf}, x_{of}) = D\left[\lambda \frac{\bar{p}}{1 - \bar{p}} \left(\frac{x_{of}}{x_{nf}}\right)^{\delta}\right] v^{of} - x_{of}. \tag{19}$$

In order to derive the equilibrium outcome of the contest game in stage 2, we now establish important properties of the above payoff functions. We start with V:

LEMMA 1 *Suppose that $x_{of} > 0$. Then, V is a function of x_{nf} on $[0, \infty)$ that has a local maximum at 0 and at most one interior local maximum.*

Proof. See Appendix B. ∎

Figure 3 shows two examples of how V can look like. In both examples, V has an interior local maximum. In panel $a.$, the fee charged by the new-economy FI is sufficiently large such that the interior maximum is also the global one. In panel $b.$, v^{nf} takes a lower value—with the result that the boundary maximum becomes the global maximum.

Lemma 1 leaves open whether the new-economy FI would enter the contest if their opponent entered it. But for the old-economy FI, the case is clear:

LEMMA 2 *Suppose that $x_{nf} > 0$. Then, W is a strictly concave function of x_{of} on $[0, \infty)$ with an interior global maximum.*

Proof. See Appendix B. ∎

The differences in the properties of the two payoff functions are due to the asymmetric nature of the exponential distribution. For any given $x_{nf} > 0$, the density function of the exponential distribution, $D'[\cdot]$, takes a large value if x_{of} is in the neighborhood of 0. So, an

16

Figure 3: The new-economy FI's payoff as a function of x_{nf}

a. High v^{nf} b. Low v^{nf}
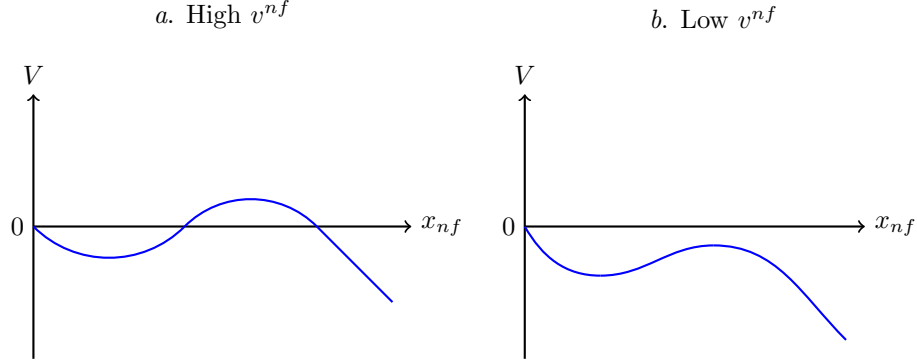
increase in x_{of} from a low level has a large positive effect on the probability that the contest is won by the old-economy financier. By contrast, for any given $x_{of} > 0$, $D'[\cdot]$ is small if x_{nf} is in the neighborhood of 0. As a result, when x_{nf} increases from a low level, the chance that the contest is won by the new-economy FI rises comparatively slowly.

We find that the new-economy FI enters the contest if the regulation is not too strict:

PROPOSITION 3 *Suppose that*

$$\lambda < \bar{\lambda} \equiv \left(\frac{v^{nf}}{v^{of}}\right)^{\delta} \frac{1-\bar{p}}{\bar{p}} \frac{1}{\delta}. \tag{20}$$

Then, in stage 2 of the game, the belief contest has a unique pure strategy equilibrium in which the campaigning efforts are given by

$$x_{nf}^* = v^{nf} \cdot \Omega\left(\delta, \lambda, \bar{p}, v^{nf}/v^{of}\right) \tag{21}$$

and

$$x_{of}^* = v^{of} \cdot \Omega\left(\delta, \lambda, \bar{p}, v^{nf}/v^{of}\right), \tag{22}$$

where

$$\Omega\left(\delta, \lambda, \bar{p}, v^{nf}/v^{of}\right) \equiv \left\{\lambda\delta\frac{\bar{p}}{1-\bar{p}}\left(\frac{v^{nf}}{v^{of}}\right)^{-\delta}\exp\left[-\lambda\frac{\bar{p}}{1-\bar{p}}\left(\frac{v^{nf}}{v^{of}}\right)^{-\delta}\right]\right\}. \tag{23}$$

From equations (21) and (22), we obtain

$$\frac{x_{nf}^*}{x_{of}^*} = \frac{v^{nf}}{v^{of}}. \tag{24}$$

Proof. See Appendix B. ∎

17

The result that in equilibrium the spending ratio equals the fee ratio entails that the equilibrium prior belief, p^*, takes a simple form. By combining equations (6), (7), and (24), we get

$$p^*(\tilde{\zeta}) = \frac{\tilde{\zeta}/\lambda}{\tilde{\zeta}/\lambda + (v^{nf}/v^{of})^{-\delta}} \in (0,1). \tag{25}$$

There are three determinants of investors' equilibrium subjective prior belief. First, it is an increasing function of v^{nf}/v^{of}. This follows from the fact that the higher this ratio, the greater the new economy FI's incentive to choose a high campaigning effort, since the expected payoff of "winning" is high. The second determinant is the random shock $\tilde{\zeta}$. This captures that the success of campaigning efforts and the associated narrative are to some degree determined by accidental forces. For instance, p^* may take a high value if the new economy FI has designed a narrative that happens to go viral due to lucky timing of a celebrity endorsement based on a catchy slogan.[13] Note that, because of the presence of $\tilde{\zeta}$, p^* is a random variable. The third determinant is the regulatory stringency λ, reflecting that a higher stringency reduces the productivity of campaigning efforts by the new-economy FI.

To summarize, provided that λ satisfies condition (20), Propositions 1 to 3, jointly with equation (25), describe the outcomes of stages 2, 3 and (possibly) 5 of the game. Those outcomes are influenced by (up to) two random forces: given x^*_{nf} and x^*_{of}, the narrative shock $\tilde{\zeta}$ determines the equilibrium prior $p^*(\tilde{\zeta})$ and hence whether or not the new economy takes off. Provided that the new economy does take off, investors' liquidation decisions are determined by the informative but noisy signal S if $p^*(\zeta) \in [\bar{p}, \sigma)$.

4.4 Three Categories of Beliefs

Equation (25) clarifies how factors such as the fee ratio, the narrative shock, and regulation affect the equilibrium subjective prior belief about the probability of $F = H$. Together with the signal's quality, σ, and the new technology's promise, α^h/β (see equation 16), those factors determine the equilibrium pattern of investment: if $p^*(\tilde{\zeta}) < \bar{p}(\alpha^h/\beta, \sigma)$, investors' capital is channeled into the established technology; otherwise, it goes into the PTI. In the latter case, any negative signal ($S = L$) would be ignored if $p^*(\tilde{\zeta}) \geq \sigma$.

Figure 4 depicts the three possible regimes that may result in equilibrium. The three shaded areas represent the probabilities of, respectively, a win of the established technology, a PTI win with the signal being adhered to later on, and a PTI win with a possible negative signal

---

[13]See Shiller (2019) for examples.

Figure 4: Three different priors and their probabilities
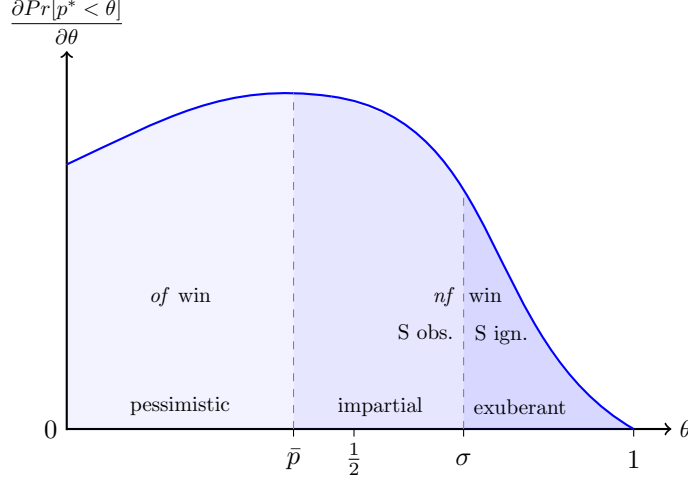
being ignored. Those probabilities can be calculated using

$$\Pr\left[p^*(\tilde{\zeta}) \geq \theta\right] = \exp\left\{-\lambda \frac{\theta}{1-\theta}\left(\frac{v^{nf}}{v^{of}}\right)^{-\delta}\right\}, \tag{26}$$

where $\theta \in (0,1)$ is an arbitrary threshold. For instance, if $\theta = \bar{p}(\alpha^h/\beta, \sigma)$, where $\bar{p}$ is given by equation (16), equation (26) gives the sum of the two areas to the right of $\bar{p}$, i.e., the probability that the investors place their investments in PTI firms. In fact, with $\theta = \bar{p}$, the equation assembles all parameters that according to the model influence whether a new and uncertain technology will be able to attract large-scale funding. The equation implies that the PTI stands a better chance when investment banks can charge relatively large fees (i.e., when v^{nf}/v^{of} is larger); when the PTI lends itself more to informative early testing (i.e., when σ is larger); when its promise is greater (i.e., when α^h/β is larger); or in the case of less stringent regulation (i.e., when λ is smaller).

Rationality as it is commonly understood in economics, and game theory in particular, does not place any restrictions on prior beliefs, but only on the updating process. In a narrow sense, investors' priors as derived above do not violate any of the standard axioms of rational choice. Nevertheless, it is interesting to ask how an investor with an uninformative prior would choose. Although the choice of any uninformed prior is to some degree arbitrary (see, e.g., Van Den Steen, 2011), a natural choice for the binary random variable F would be the principle of indifference (see, e.g., Gilboa, 2009, p. 17-19). In the current context, the principle of indifference would entail that an investor treats p as a realization of a random variable P

19

with a uniform distribution on $[0, 1]$. As an investor committed to the principle of indifference would not view any particular p to be more likely than any other, we call such an investor "impartial". How would an impartial investor decide?

PROPOSITION 4 *Suppose that an impartial investor is one that applies the principle of indifference with respect to p. An impartial investor opts for the new economy in stage 3 and follows signal S when deciding on continuation/liquidation in stage 5.*

Proof. See Appendix B. ∎

The notion of a hypothetical impartial investor is useful as a benchmark that helps us categorize the subjective prior beliefs held by the investors populating our economy.[14] In terms of investment behavior in our model, there is no deviation from the impartial-investor benchmark as long as $p^*(\tilde{\zeta}) \in [\bar{p}, \sigma)$. In that case, investors opt for the new economy in stage 3 and then adhere to signal S in stage 5. As a result, we characterize a subjective prior that falls into this medium range as *impartial*. However, if either $p^*(\tilde{\zeta}) < \bar{p}$ or $p^*(\tilde{\zeta}) \geq \sigma$, the investors' behavior differs from the impartial benchmark: in the former case, investors always prefer the old over the new economy, while in the latter case investors fail to liquidate PTI investments if $S = L$. Accordingly, subjective priors that fall into the range $(0, \bar{p})$ are characterized as *pessimistic* and those that fall into the range $[\sigma, 1)$ as *exuberant*. If investors are pessimistic, they do not pursue any PTI investment opportunities and thus fail to produce objective information on state F; exuberance, on the other hand, means that investors do not hesitate to disregard the only source of objective information regarding the fundamental state. Broadening the perspective, pessimistic priors arguably have a negative consequence for innovation-based growth, while exuberant beliefs may lead to an increased likelihood of boom and bust cycles, as we discuss next.

4.5 Relating the Model to Evidence on Boom and Bust Episodes

From the model, we obtain predictions that can be related to empirical observations on boom and bust episodes. To this effect, we observe that—unless the true probability of $F = H$ equals one—a possible equilibrium outcome of the game is that even beyond stage 5 capital is allocated to a PTI that does not live up to its promise. This is either the case if $F = L$ and investors hold an impartial prior but are fooled by a wrong signal ($S = H \neq F$), or if $F = L$ and investors hold an exuberant prior ($p^*(\tilde{\zeta}) \geq \sigma$). Although we do not explicitly model

---

[14]In fact, the investment behavior described in Proposition 4 does not require a uniform prior. Other symmetric distributions, e.g., Jeffreys Prior Beta(1/2, 1/2), would lead to exactly the same behavior.

Figure 5: Signal quality and boom and bust episodes—model prediction
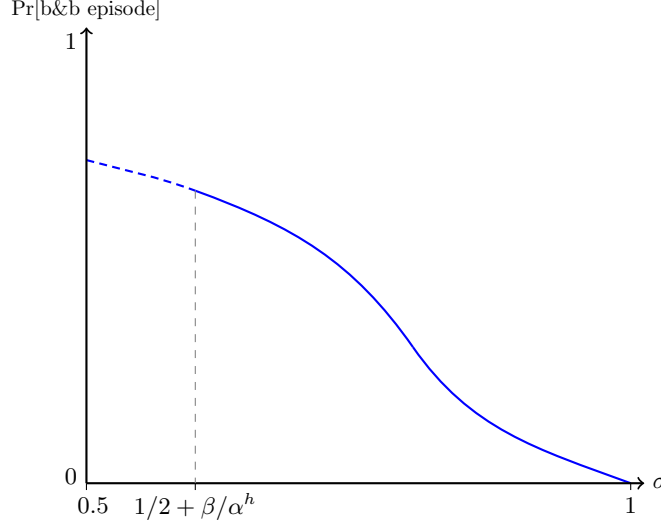
how the game would continue after the materialization of payoffs, it is natural to assume that disappointed PTI investors would withdraw their capital from the new economy. In other words: the model suggests the occurrence of boom and bust (b&b) episodes, with the new technology attracting capital in stage 3 and then losing it again after the materialization of payoffs. Using π for the unknown true probability of $F = H$, the probability with which a boom and bust episode emerges in equilibrium can be written as

$$\Pr\left[\text{b\&b episode}\right] = (1 - \pi)\left\{\sigma \Pr\left[p^*(\tilde{\zeta}) \geq \sigma\right] + (1 - \sigma)\Pr\left[p^*(\tilde{\zeta}) \geq \bar{p}\right]\right\}, \qquad (27)$$

where the two probabilities on the right-hand side of equation (27) are specified in equation (26).

From equation (27), we can derive comparative-static results with respect to signal quality, σ, and the fee ratio, v^{nf}/v^{of}. For all $\pi < 1$, the probability of a boom and bust episode is a monotonically decreasing function of σ, as is shown in Figure 5: if σ rises, both the realization of a wrong signal and the adoption of an exuberant subjective prior become less likely.[15] The predicted negative relationship between signal quality and the probability of a boom and bust episode can be taken to the data. Empirical proxies for those two magnitudes can be constructed from information in Goldfarb and Kirsch (2019), an extensive study that traces major technological innovations over the past 180 years. First, consider the proxy for signal quality. For each technology, Goldfarb and Kirsch (2019) specify when it first emerged

---

[15]Figure 5 assumes a relatively low value for π, so that boom and bust episodes are relatively likely if σ is low. The dashed section on the left refers to values of σ where condition (R1) does not hold.

Figure 6: Signal quality and boom and bust episodes—data
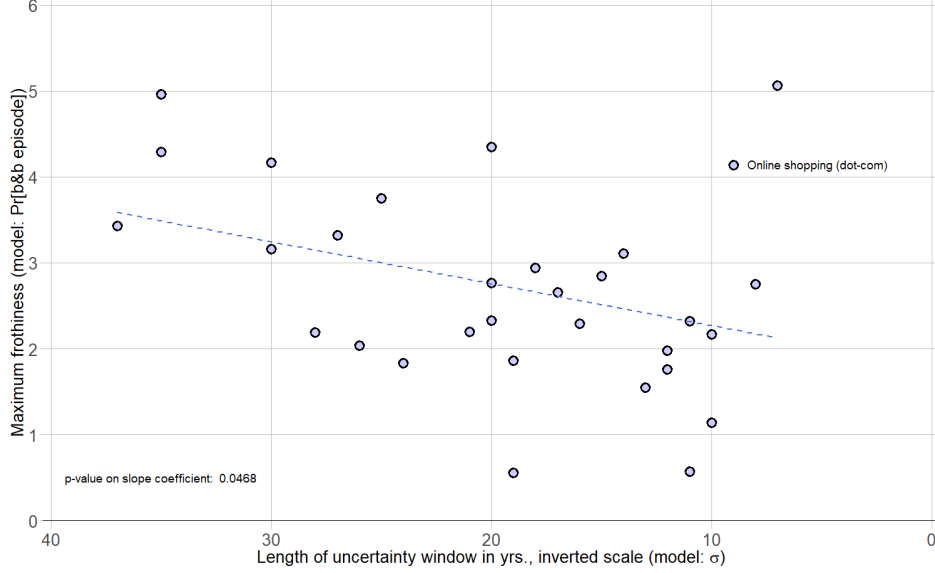
Notes: This scatter plot uses data from Goldfarb and Kirsch (2019), Tables A.1 and A.2 (downloaded on June 18, 2021, from the book's website).

and when the major uncertainties were resolved. From this information, we can calculate the length of the uncertainty window in years. A long uncertainty window in practice corresponds to a low signal quality in the model: low quality means that with the arrival of the signal in stage 4 uncertainty is not much reduced and thus extends into stage 5; with a high signal quality, uncertainty essentially gets eliminated in stage 4. To find a proxy for the probability of a boom and bust episode, we exploit that Goldfarb and Kirsch (2019) screen the stock prices of the companies involved in a new technology for "frothiness". Frothiness is defined in terms of standard deviations from the trend, "where the 'trend' is the predicted stock price looking forward and backward seven years." (p. 35). Peak frothiness is then defined to be the highest frothiness level within the uncertainty window. Following the authors, we interpret a large peak-frothiness level as an indication of a boom and bust episode. Therefore, peak frothiness is our empirical proxy for the probability of such an episode in the model.

In total, there are 31 technological innovations for which Goldfarb and Kirsch (2019) provide data on both the uncertainty window and peak frothiness. Figure 6 plots peak frothiness against the length of the window of uncertainty. In the figure, the horizontal axis uses an inverted scale, so that window length decreases from left to right (remember: a long window corresponds to low signal quality and a short window to a high quality). The empirical pattern in Figure 6 is

22

consistent with the model prediction in Figure 5: a shorter uncertainty window (model: larger σ) is associated with lower peak frothiness (model: smaller $\Pr[\text{b\&b episode}]$). Despite the low number of observations, the correlation is statistically significant (5% level).

While signal quality and the probability of a boom and bust episode are negatively related, we have a positive relationship between v^{nf}/v^{of} and $\Pr[\text{b\&b episode}]$: with a higher fee ratio, the imbalance of campaigning efforts in favor of the new technology is larger—which on average means more friendly beliefs towards the innovation. Also, this second comparative-static prediction appears consistent with the facts. During the two most recent major boom and bust episodes, dot-com in the 1990s and securitization of residential mortgages in the 2000s, the fees charged by the respective new-economy FIs were exceptionally large.[16]

5 Analysis: Regulator

5.1 Regulation without Objective information

The equilibrium beliefs stated in equation (25), as well as the probabilities of the three belief categories discussed in Section 4.4 (see equation 26) depend on the parameter λ. We have associated this parameter with regulatory stringency in the sense that more stringency makes it harder for the new economy FI to win the belief contest and to instill an exuberant belief. Given this link, it is natural to ask what would be a "good" value of λ for the regulator to be set. Addressing this question requires assumptions as to what the regulator knows, believes, and wishes to achieve. Regarding knowledge, we want to put forth a framework in which the regulator has no more objective information on state F than the other agents. However, in contrast to the investors, the regulator does not adopt any subjective prior about $\Pr[F = H]$. Along the lines of, e.g., Brunnermeier et al. (2014) and Buss et al. (2016), we assume that the regulator understands that the investors' prior belief p^* in equation (25) is purely subjective and does not rely on any objective information. This asymmetry may reflect that processes for selecting people into positions with regulatory powers often involve various parties with diverse views—and so tend to favor neutral contenders.

Yet, without superior information, and knowing that the belief held by the investors has no objective base, the regulator lacks a well-defined social welfare function. As a result, determining λ based on social welfare is infeasible. In what follows, we argue that the regulator might want to follow a heuristic yet feasible approach that is not rooted in the maximization

---

[16]See Subsection 5.2 below and, e.g., Tett (2009, p. 122) and Fligstein and Roehrkasse (2016, p. 618) concerning the role and magnitude of fees in the dot-com and the securitization episode, respectively.

of a social welfare function but builds on the logic of the three belief categories discussed in Subsection 4.4. For the regulator, a natural objective would be to choose λ so as to maximize the probability that investors adopt an impartial prior belief. In pursuing this objective, the regulator—taking the structural parameters of the economy into account—would address the risk of "investor exuberance" without choking off the opportunity to discover ever more productive technologies; at the same time, the regulator would not be required to adopt an own subjective prior belief about $\Pr[F = H]$. Given these considerations, we propose that the regulator's objective function is given by $\Pr\left[\bar{p} \leq p^*(\tilde{\zeta}) < \sigma\right]$. Using equation (26), we get

$$\Pr\left[\bar{p} \leq p^*(\tilde{\zeta}) < \sigma\right] = \exp\left\{-\lambda \frac{\bar{p}}{1-\bar{p}}\left(\frac{v^{nf}}{v^{of}}\right)^{-\delta}\right\} - \exp\left\{-\lambda \frac{\sigma}{1-\sigma}\left(\frac{v^{nf}}{v^{of}}\right)^{-\delta}\right\}. \qquad (28)$$

The maximization of objective function (28) pins down the optimal level of regulation (according to this heuristic framework):

PROPOSITION 5 *Suppose that the regulator wants to maximize the probability that the investors adopt an impartial prior belief in stage 3 of the game (given by equation 28). Then, the regulator opts for*

$$\lambda^* = \left(\frac{v^{nf}}{v^{of}}\right)^{\delta} \frac{\ln\left(\frac{\sigma}{1-\sigma}\right) - \ln\left(\frac{\bar{p}}{1-\bar{p}}\right)}{\frac{\sigma}{1-\sigma} - \frac{\bar{p}}{1-\bar{p}}} < \bar{\lambda}, \qquad (29)$$

where $\bar{p}$ and $\bar{\lambda}$ are given by equations (16) and (20), respectively.

Proof. See Appendix B. ■

When choosing λ, the regulator faces a trade-off. A low level of λ means that the probability of investors adopting a pessimistic prior is small (desired), while the chance that possible PTI investors will disregard a negative signal—i.e., adopt an exuberant prior—is large (undesired). With a large λ, the situation is reversed. Figure 7 illustrates that this trade-off gives rise to a hump-shaped (quasi-concave) relationship between the probability of investors adopting an impartial prior and the stringency of regulation. The key structural parameters of the model affect the location of the peak, λ^*, as follows:

PROPOSITION 6 *For the level of regulatory stringency characterized in Proposition 5, λ^*, we obtain the following comparative-static results:*

$$\frac{\partial \lambda^*}{\partial (v^{nf}/v^{of})} > 0, \quad \frac{\partial \lambda^*}{\partial \sigma} < 0, \quad \frac{\partial \lambda^*}{\partial (\alpha^h/\beta)} > 0. \qquad (30)$$

Proof. See Appendix B. ■

Figure 7: The chance of investors adopting an impartial prior as a function of λ
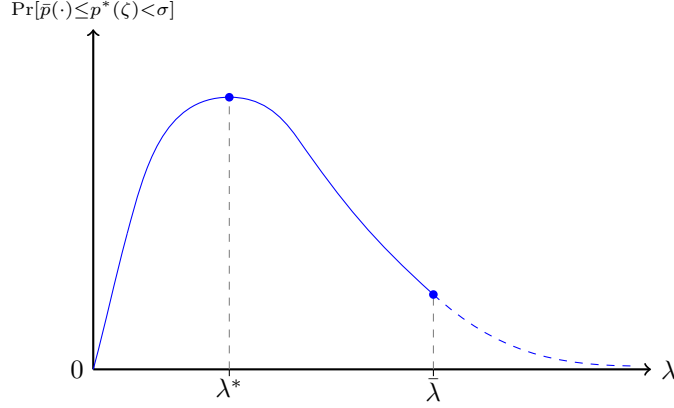
Stricter regulation increases the probability of a pessimistic prior (undesired) and reduces the probability of an exuberant one (desired). When v^{nf}/v^{of} rises (i.e., when the new economy FI has a comparatively greater incentive to put more effort into the contest), the former (undesired) effect weakens in relative terms. By raising λ, the regulator thus can lower the risk of exuberant investment without facing an equivalent increase in the probability of pessimistic investment. A similar logic applies when α^h/β rises (i.e., when the new technology becomes more promising). Finally, when σ rises (i.e., when the signal's quality improves), the effect of stricter regulation on the risk of exuberant investment falls in relative terms. As a consequence, the regulator should relent: the probability of pessimistic investment can be reduced without giving rise to an equivalent increase in the risk of exuberant investment.

5.2   Back to Dot-Com from a Regulation Perspective

If at the beginning of the dot-com boom in the mid-1990s a regulator had examined empirical proxies for the three magnitudes considered in Proposition 6, what pattern would have emerged? And what would have been the conclusions for regulatory stringency? First, consider the fee ratio, v^{nf}/v^{of}, which in practice may correspond to the ratio of fees earned in IPO and bond underwriting, respectively. In the 1990s, according to Chen and Ritter (2000, p. 1105), "the IPO business [was] very profitable," partly due to implicit collusion among IPO underwriters.[17] On the other hand, Gande et al. (1999) show that, due to increased competition, the fees in bond underwriting had come down significantly. These facts suggest that in the

---

[17]This is consistent with the observation by Tett (2009, p. 88) that dot-com IPOs "delivered a stunning stream of revenues (...)." High underwriting fees were not the only source of profits. Kindleberger and Aliber (2015) note that the practice of short-run underpricing of IPOs was an additional important form of compensation.

mid-1990s the fee ratio had reached an unusually high level.

The model parameter for signal quality, σ, may in practice correspond to the degree to which an innovation is suitable for early testing and small-scale experimentation. In the case of online businesses, serious obstacles to testing and experimentation—such as network effects—were largely absent. In fact, Morgan Stanley's "Internet Report" (Meeker and DePuy, 1996), a highly influential study on the commercialization of the Internet published in early 1996, highlighted the potential for experimentation with well-defined market segments and product categories. This assessment is consistent with information from Goldfarb and Kirsch (2019), according to which the window of uncertainty associated with the innovation "online shopping" was very short (see Section 4.5 above). All this suggests that in the mid-1990s the regulator could have acted on the assumption of a high signal quality.

The innovation's promise, α^h/β, is the final magnitude considered in Proposition 6. In the case at hand, an appropriate real-world proxy might have been the medium-term profitability of online businesses (relative to that of ordinary retailers) if *in fact* the number of customers quickly switching to online shopping reached the upper end of the possible spectrum. It was clear to the contemporary observer that in this positive event (whose probability of occurrence was of course the great unknown) medium-term profitability would be very high: at the time, market research companies (e.g., International Data Corporation) published reports forecasting a huge market potential,[18] while sometimes also outlining the case for cost savings (e.g., due to lower physical capital requirements or a greater scope for automated transactions). So, in the mid-1990s, the regulator would have rated the innovation's promise to be great.

To summarize, from the perspective of the mid-1990s, two out of the three guiding variables identified here clearly would have called for stringency, while just one would have spoken in favor of loose rules. Therefore, overall, the model's advice would have been that in view of the emerging dot-com boom, the regulator ponder a tightening of the regulations in the field of financial consumer protection—notwithstanding the general trend towards liberalization.

6   Conclusion

We provide a model of how investors form subjective prior beliefs about a potentially transformative innovation (PTI) that comes with a great yet—from an objective view—highly uncertain promise. In the model, investors adopt subjective prior beliefs that arise from a belief contest

---

[18]In an article on a contemporaneous International Data Corporation report, Reuters (Dec 1, 1997) wrote that "the Internet is expected to launch an economic quantum shift."

fought by competing financial intermediaries with incentives to steer capital either towards or away from the PTI. We show how looking at belief formation through the prism of a belief contest can guide a financial regulator who is about to re-calibrate the stringency of (financial consumer protection) regulation in view of an uncertain innovation that possibly attracts large amounts of capital. We argue that the regulator may want to maximize the chance that investors adopt an impartial prior that makes them heed objective information that is expected to emerge over time; and we clarify what kind of a priori collectible data the regulator needs to gather and ponder to pursue this goal. Essentially, we sketch at a conceptual level how regulation can be theory- and data-driven even in the limiting case of absent objective information on the success chances of a PTI. While we chose to keep the current setup clean and simple, future work can extend it to include features such as risk aversion, heterogeneity in subjective beliefs, or the availability of a small piece of objective information prior to the belief contest.

References

Albright, R. E. (2002). What can past technology forecasts tell us about the future? *Technological Forecasting and Social Change*, 69(5):443–464.

Ben-David, I., Graham, J. R., and Harvey, C. R. (2013). Managerial miscalibration. *The Quarterly journal of economics*, 128(4):1547–1584.

Bénabou, R., Falk, A., and Tirole, J. (2020). Narratives, Imperatives, and Moral Persuasion. Working Paper; previous version available as NBER Working Paper 24798.

Bergstresser, D., Chalmers, J. M., and Tufano, P. (2008). Assessing the costs and benefits of brokers in the mutual fund industry. *The Review of Financial Studies*, 22(10):4129–4156.

Binswanger, J. and Oechslin, M. (2021). The economics of beliefs under fundamental uncertainty. Working Paper.

Bolton, P., Freixas, X., and Shapiro, J. (2007). Conflicts of interest, information provision, and competition in the financial services industry. *Journal of Financial Economics*, 85:297–330.

Brunnermeier, M. K., Simsek, A., and Xiong, W. (2014). A welfare criterion for models with distorted beliefs. *Quarterly Journal of Economics*, 129(4):1753–1797.

Buss, A., Dumas, B., Uppal, R., and Vilkov, G. (2016). The intended and unintended consequences of financial-market regulations: A general-equilibrium analysis. *Journal of Monetary Economics*, 81:25–43.

Cassidy, J. (2003). The Investigation. *New Yorker*.

Chen, H. C. and Ritter, J. R. (2000). The seven percent solution. *Journal of Finance*, 55(3):1105–1131.

Cho, I.-K. and Kreps, D. M. (1987). Signaling games and stable equilibria. *The Quarterly Journal of Economics*, 102(2):179–221.

Crawford, V. P. and Sobel, J. (1982). Strategic information transmission. *Econometrica*, 50(6):1431–1451.

David, P. A. (1990). The dynamo and the computer: An historical perspective on the modern productivity paradox. *The American Economic Review*, 80(2):355–361.

Egan, M. (2019). Brokers versus Retail Investors: Conflicting Interests and Dominated Products. *Journal of Finance*, LXXIV(3):1217–1260.

Fligstein, N. and Roehrkasse, A. F. (2016). The Causes of Fraud in the Financial Crisis of 2007 to 2009: Evidence from the Mortgage-Backed Securities Industry. *American Sociological Review*, 81(4):617–643.

Gaganis, C., Galariotis, E., Pasiouras, F., and Staikouras, C. (2020). Bank profit efficiency and financial consumer protection policies. *Journal of Business Research*, 118:98–116.

Gande, A., Puri, M., and Saunders, A. (1999). Bank entry, competition, and the market for corporate securities underwriting. *Journal of Financial Economics*, 54(2):165–195.

Gilboa, I. (2009). *Theory of decision under uncertainty.* Cambridge University Press, New York.

Gilboa, I., Samuelson, L., and Schmeidler, D. (2014). No-Betting-Pareto Dominance. *Econometrica*, 82(4):1405–1442.

Goldfarb, B. and Kirsch, D. A. (2019). *Bubbles and Crashes. The Boom and Bust of Technological Innovation.* Stanford University Press, Stanford.

Graves, S. B. and Ringuest, J. (2018). Overconfidence and disappointment in venture capital decision making: An empirical examination. *Managerial and Decision Economics*, 39(5):592–600.

Hirshleifer, D., Low, A., and Teoh, S. H. (2012). Are overconfident CEOs better innovators? *The Journal of Finance*, 67(4):1457–1498.

Hirshleifer, J. (1995). Anarchy and its Breakdown. *Journal of Political Economy*, 103(1):26–52.

Hoechle, D., Ruenzi, S., Schaub, N., and Schmid, M. (2018). Financial advice and bank profits. *The Review of Financial Studies*, 31(11):4447–4492.

Kahneman, D. (2003). Maps of bounded rationality: Psychology for behavioral economics. *American Economic Review*, 93(5):1449–1475.

Kahneman, D. (2011). *Thinking, Fast and Slow.* Farrar, Straus and Giroux, New York.

Kamenica, E. (2019). Bayesian persuasion and information design. *Annual Review of Economics*, 11:249–272.

Kamenica, E. and Gentzkow, M. (2011). Bayesian persuasion. *American Economic Review*, 101(6):2590–2615.

Kaustia, M. and Perttula, M. (2012). Overconfidence and debiasing in the financial industry. *Review of Behavioural Finance*.

Kindleberger, C. P. and Aliber, R. Z. (2015). *Manias, panics and crashes*. Palgrave Macmillan, New York, 7 edition.

Martens, M. L., Jennings, J. E., and Jennings, P. D. (2007). Do the stories they tell get them the money they need? the role of entrepreneurial narratives in resource acquisition. *Academy of Management Journal*, 50(5):1107–1132.

McKenzie, C. R., Liersch, M. J., and Yaniv, I. (2008). Overconfidence in interval estimates: What does expertise buy you? *Organizational Behavior and Human Decision Processes*, 107(2):179–191.

Meeker, M. and DePuy, C. (1996). *Morgan Stanley. The Internet Report*. Harper Business, New York.

Mokyr, J. (2013). Cultural entrepreneurs and the origins of modern economic growth. *Scandinavian Economic History Review*, 61(1):1–33.

Mokyr, J. (2016). *A culture of growth: the origins of the modern economy*. Princeton University Press, Princeton.

Mullainathan, S., Noeth, M., and Schoar, A. (2012). The market for financial advice: An audit study. NBER Working Paper 17929.

Savage, T., Davis, A., Fischhoff, B., and Morgan, M. G. (2021). A strategy to improve expert technology forecasts. *Proceedings of the National Academy of Sciences*, 118(21).

Shiller, R. J. (2002). Bubbles, Human Judgment, and Expert Opinion. *Financial Analysts Journal*, 58(3):18–26.

Shiller, R. J. (2017). Narrative economics. *American Economic Review*, 107(4):967–1004.

Shiller, R. J. (2019). *Narrative economics: How stories go viral and drive major economic events*. Princeton University Press.

Spence, M. (1973). Job market signaling. *The Quarterly Journal of Economics*, 87(3):355–374.

Stoughton, N. M., Wu, Y., and Zechner, J. (2011). Intermediated Investment Management. *Journal of Finance*, LXVI(3):947–980.

Sydsæter, K. and Hammond, P. J. (2008). *Essential mathematics for economic analysis*. Pearson Education, Harlow.

Szidarovszky, F. and Okuguchi, K. (1997). On the existence and uniqueness of pure nash equilibrium in rent-seeking games. *Games and Economic Behavior*, 18(1):135–140.

Tett, G. (2009). *Fool's Gold: How Unrestrained Greed Corrupted a Dream, Shattered Global Markets and Unleashed a Catastrophe*. Hachette UK.

Tversky, A. and Kahneman, D. (1974). Judgment under uncertainty: Heuristics and biases. *Science*, 185(4157):1124–1131.

Van Den Steen, E. (2011). Overconfidence by Bayesian-rational agents. *Management Science*, 57(5):884–896.

Appendix A: Technology

New Economy

The technology characterizing the representative PTI investment opportunity $i \in [0, 1]$ is represented by a linear production function with a minimum investment requirement:

$$y_i = f^n(k_i) = \begin{cases} \bar{\alpha} + \alpha^h k_i & : & k_i \geq \theta^n \\ 0 & : & k_i < \theta^n \end{cases} , \tag{31}$$

where y_i and k_i refer to output and capital invested, respectively, and $\theta^n \in (0, 1)$ is the minimum investment threshold. We assume $\bar{\alpha} > 0$ and $\alpha^h \geq \beta > 0$ (as noted in Subsection 3.1). Moreover, suppose that the depreciation rate that applies to new-economy investment opportunities, $\kappa^n \geq 0$, depends on state $F \in \{H, L\}$ as follows:

$$\kappa^n(F) = \begin{cases} 0 & : & F = H \\ 2\alpha^h & : & F = L \end{cases} . \tag{32}$$

Under the assumption that new-economy investment opportunities let investors (who can be understood as the new stock owners) earn the marginal product of capital net of depreciation, and provided that $k_i \geq \theta^n$, equations (31) and (32) imply that investors who get involved in opportunity i earn a rate of return as specified by equation (1). In this regard, observe that equation (1) relies on the definition $\alpha^l \equiv \alpha^h - \kappa^n(L) = -\alpha^h$.

Given these assumptions, opportunity i's earnings after the cost of capital, z_i, are given by

$$z_i = g^n(k_i) = \begin{cases} f^n(k_i) - \alpha^h k_i = \bar{\alpha} & : & k_i \geq \theta^n \\ -\alpha^h k_i & : & k_i < \theta^n \end{cases} . \tag{33}$$

So, when it comes to earnings after the cost of capital, the level of k_i matters only inasmuch as it must exceed θ^n; if this condition is met, z_i is equal to $\bar{\alpha}$ and hence independent of k_i. In turn, this implies that the original owner of investment opportunity i (who can be understood as a venture capitalist) is indifferent as to the exact level of capital raised. We can interpret $\bar{\alpha}$ as the reward to the production factor "entrepreneurship".

Finally, we assume that the fee charged by the new economy FI, v^{nf}, is borne by the venture capitalist. There is an upper bound to this fee: it must not exceed the expected reward for entrepreneurship, i.e., $\bar{\alpha}$ times the venture capitalist's subjective probability that the investors will not choose liquidation in stage 5 (liquidation means that firm output, and hence the reward

to entrepreneurship, are zero). If we made an assumption about the venture capitalist's own prior belief about the chance of $F = H$, the equilibrium value of this subjective probability could be calculated using the results in Section 4.

Old Economy

The technology characterizing the representative old-economy investment opportunity $j \in [0, 1]$ is represented by a production function that is of the very same type as that of the new economy:

$$
y_j = f^o(k_j) = \begin{cases} \bar{\beta} + \beta k_j & : & k_j \geq \theta^o \\ 0 & : & k_j < \theta^o \end{cases}, \tag{34}
$$

where—again—y_j and k_j denote output and capital invested, respectively, and $\theta^o \in (0, 1)$ is a minimum investment threshold. Like in the new-economy case, we assume $\bar{\beta} > 0$ and $0 < \beta \leq \alpha^h$ (as noted in Subsection 3.1). However, unlike in the new economy, the old-economy depreciation rate is invariable and normalized to zero. So, provided that $k_i \geq \theta^o$, and under the assumption that old-economy investments let investors (who can be understood as bond holders) earn the marginal product of capital, we get $r_o = \beta$, as specified by equation (5).

Given these assumptions, opportunity j's earnings after the cost of capital, z_j, are given by

$$
z_j = g^o(k_j) = \begin{cases} f^o(k_j) - \beta k_j = \bar{\beta} & : & k_j \geq \theta^o \\ -\beta k_j & : & k_j < \theta^o \end{cases}. \tag{35}
$$

As in the new economy, z_j is independent of k_j if the latter exceeds θ^o; as a result, provided that the θ^o-threshold is met, the owner of j is indifferent as to the exact level of capital employed. Again just as in the new economy, we assume that the fee charged by the old economy FI, v^{of}, is borne by the owner. Obviously, we must have $v^{of} < \bar{\beta}$.

Appendix B: Proofs

Propositions

Proof of Proposition 3. We first show that neither of the financial intermediaries has a unilateral incentive to deviate if $(x_{nf}, x_{of}) = (x^*_{nf}, x^*_{of})$. Observing equation (24), it is straightforward to check that x^*_{nf} and x^*_{of} simultaneously satisfy the first-order conditions $V_{x_{nf}}(x^*_{nf}, x^*_{of}) = 0$ and $W_{x_{of}}(x^*_{nf}, x^*_{of}) = 0$, where $V_{x_{nf}}$ and $W_{x_{of}}$ are given by equations (54) and (57), respectively. From this, and from the properties of W established in Lemma 2, it

immediately follows that x_{of}^* is the global maximum of $W(x_{nf}^*, x_{of})$ on $[0, \infty)$. As a result, provided that $x_{nf} = x_{nf}^*$, the old economy FI does not have any incentive to deviate from x_{of}^*.

Turning to the new economy FI, we note that x_{nf}^* is a candidate for the global maximum of V. We now proceed in two steps: first, we establish that x_{nf}^* is an interior local maximum; second, we prove that it is the global maximum, too. To see that x_{nf}^* is an interior local maximum, observe that, together, restriction (20) and equation (24) imply

$$\frac{x_{nf}^*}{x_{of}^*} > \left(\lambda \delta \frac{\bar{p}}{1 - \bar{p}} \right)^{1/\delta}. \tag{36}$$

Given this, we conclude from equations (55) and (56) that the second-order condition for an interior local maximum is met: $V_{x_{nf}, x_{nf}}(x_{nf}^*, x_{of}^*) < 0$. Lemma 1 establishes that V does have a boundary maximum at 0, but also that there cannot be any further interior maxima. Therefore, and because $V(0, x_{of}) = 0$ for any $x_{of} > 0$ (equation 18), proving that $V(x_{nf}^*, x_{of}^*) > 0$ means proving that x_{nf}^* is the global maximum of V. Using $x_{nf}^*/x_{of}^* = v^{nf}/v^{of}$ (equation 24) and the explicit expression for x_{nf}^* (equation 21) in equation (18), we obtain

$$V(x_{nf}^*, x_{of}^*) = v^{nf} e^{-\lambda \frac{\bar{p}}{1 - \bar{p}} \left(\frac{v^{of}}{v^{nf}} \right)^\delta} \left[1 - \delta \lambda \frac{\bar{p}}{1 - \bar{p}} \left(\frac{v^{of}}{v^{nf}} \right)^\delta \right]. \tag{37}$$

As restriction (20) guarantees that $[\cdot]$ is strictly positive, $V(x_{nf}^*, x_{of}^*) > 0$ follows. So x_{nf}^* is indeed the global maximum of $V(x_{nf}, x_{of}^*)$ on $[0, \infty)$. As a result, provided that $x_{of} = x_{of}^*$, the new economy FI does not have any incentive to deviate from x_{nf}^*.

To prove uniqueness, we first observe that there cannot be any other equilibrium in which both FIs choose strictly positive input levels. To see this, we assume that there does exist such an equilibrium (with $x_{nf}^{**} > 0$ and $x_{of}^{**} > 0$) and lead this assumption to a contradiction. To begin with, suppose that both input levels satisfy the corresponding first-order conditions stated in the first paragraph of this proof. Then, we immediately obtain $x_{nf}^{**}/x_{of}^{**} = v^{nf}/v^{of}$ and hence $x_{nf}^{**} = x_{nf}^*$ and $x_{of}^{**} = x_{of}^*$. Now suppose that x_{nf}^{**} satisfies the corresponding first-order condition, but x_{of}^{**} does not. Then, provided $x_{nf} = x_{nf}^{**}$, the old economy FI has an incentive to deviate from x_{of}^{**}. In the reverse case, it is the new economy FI that can improve their payoff by deviating unilaterally. Finally, if both first-order conditions are violated, both FIs have a unilateral incentive to deviate. So we can rule out that there exists an equilibrium other than that described in the proposition in which $x_{nf}, x_{of} > 0$.

We now rule out the existence of equilibria in which at least one FI chooses an input level of 0. First consider $x_{nf} = 0$ and $x_{of} > 0$. For this constellation, equation (57) implies that

$W_{x_{of}}$ approaches -1 when $x_{nf} = 0$ approaches 0. So the old economy FI has a unilateral incentive to deviate. Consider now $x_{nf} > 0$ and $x_{of} = 0$. Since $\lim_{x_{of} \to 0} W_{x_{of}} = \infty$ (Lemma 2), the old economy FI again has a unilateral incentive to deviate. Finally, if $x_{nf} = x_{of} = 0$, we have $p(0,0) = 1/2$. Given this, the old economy FI has a unilateral incentive to deviate: since $\bar{p} < 1/2$ (equation 16), $p = 1/2$ implies a new economy win; yet the old economy FI can secure $p = 0$ by marginally raising x_{of}. So equilibria in which at least one FI chooses an input level of 0 cannot exist. ∎

Proof of Proposition 4. We denote by $g_P(p) = 1$ the prior density of $P \sim U[0,1]$ and by $g_{P|S=X}(p)$ the posterior density of P, given $S = X \in \{H, L\}$. Standard calculations lead to

$$g_{P|S=H}(p) = 2(1 - \sigma) + (4\sigma - 2)p \tag{38}$$

and

$$g_{P|S=L}(p) = 2\sigma - (4\sigma - 2)p. \tag{39}$$

Now consider an impartial investor who, after having received signal S in stage 4, has to decide on whether or not to liquidate their new-economy investment in stage 5. To do so, they calculate the expected rate of return under the appropriate posterior density of P. This calculation is done in two steps. The first step is to calculate the expected rate of return under a particular posterior belief $q(S, p)$. We denote this expectation by $\mathbb{E}_{q(S,p)}[R_n]$, where $S \in \{H, L\}$ and $p \in [0, 1]$. The second step then is to calculate the average expectation over all $p \in [0, 1]$, given signal S. This yields the magnitude the impartial investors is interested in: the expected rate of return under the appropriate posterior density $g_{P|S=X}(p)$, where $X \in \{H, L\}$. We denote this expectation by $\mathbb{E}_{g_{P|S=X}}[R_n]$.

First suppose that $S = H$. Using equation (10), and taking account of $\alpha^l = -\alpha^h$, we obtain

$$\mathbb{E}_{q(H,p)}[R_n] = \frac{p\sigma}{p\sigma + (1-p)(1-\sigma)}\alpha^h + \frac{(1-p)(1-\sigma)}{p\sigma + (1-p)(1-\sigma)}(-\alpha^h). \tag{40}$$

Given this, we find that

$$\mathbb{E}_{g_{P|S=H}}[R_n] = \int_0^1 \mathbb{E}_{q(H,p)}[R_n] \cdot g_{P|S=H}(p) \, dp = 2\alpha^h(\sigma - 1/2) > 0. \tag{41}$$

Note that $\mathbb{E}_{g_{P|S=H}}[R_n]$ is strictly positive because $\sigma > 1/2$ (informative signal). The corre-

sponding expression for the case $S = L$ is given by

$$\mathbb{E}_{g_{P|S=L}}[R_n] = \int_0^1 \mathbb{E}_{q(L,p)}[R_n] \cdot g_{P|S=L}(p) \; dp = -2\alpha^h(\sigma - 1/2) < 0. \tag{42}$$

Given equations (41) and (42), the impartial investor's continuation/liquidation decision in stage 5 is quickly determined. Since liquidation means a rate of return of 0, the new-economy investment is continued if $S = H$ and liquidated if $S = L$.

In stage 3, when deciding whether or not to invest in the new economy, the impartial investor anticipates their behavior in stage 5. For this behavior, the expected rate of return under a particular prior p is given by equation (13). The impartial investor calculates the average expectation over all $p \in [0, 1]$, using prior density $g_P(p) = 1$:

$$\mathbb{E}_{g_P}[R_n] = \int_0^1 \mathbb{E}_p[R_n] \cdot g_P(p) \; dp = \alpha^h(\sigma - 1/2). \tag{43}$$

Restriction (R1) guarantees that $\mathbb{E}_{g_P}[R_n] > \beta$. So, to summarize, the impartial investor opts for the new economy in stage 3 and heeds signal S in stage 5. $\blacksquare$

Proof of Proposition 5. In what follows, it is convenient to use the definitions

$$a \equiv \frac{\sigma}{1 - \sigma} \quad \text{and} \quad b \equiv \frac{\bar{p}(\cdot)}{1 - \bar{p}(\cdot)}.$$

$\Pr\left[\bar{p}(\cdot) \leq p^*(\tilde{\zeta}) < \sigma\right]$ is given in equation (28). The first derivative with respect to λ reads

$$\frac{d\Pr\left[\bar{p}(\cdot) \leq p^*(\tilde{\zeta}) < \sigma\right]}{d\lambda} = -b\left(\frac{v^{nf}}{v^{of}}\right)^{-\delta} e^{-\lambda b\left(\frac{v^{nf}}{v^{of}}\right)^{-\delta}} + a\left(\frac{v^{nf}}{v^{of}}\right)^{-\delta} e^{-\lambda a\left(\frac{v^{nf}}{v^{of}}\right)^{-\delta}}. \tag{44}$$

Imposing that this derivative be 0 (first-order condition) pins down a unique λ, the one that is stated in the proposition: λ^*. Before checking the second-order condition, we establish that $\lambda^* < \bar{\lambda}$. Using the expressions for $\bar{\lambda}$ and λ^* given in equations (20) and (29), respectively, we find that the latter condition is equivalent to

$$\frac{\ln(a) - \ln(b)}{a - b} < \frac{1}{\delta b}. \tag{45}$$

Rearranging terms yields

$$\ln\left(\frac{a}{b}\right) < \frac{1}{\delta}\left(\frac{a}{b} - 1\right). \tag{46}$$

Observe that $a/b > 1$ because $\sigma > \bar{p}$. Taking this into account, and noting that $1/\delta > 1$, it follows that condition (46) is satisfied. As a result, $\lambda^* < \bar{\lambda}$ holds indeed.

The second derivative of $\Pr\left[\bar{p}(\cdot) \le p^*(\tilde{\zeta}) < \sigma\right]$ with respect to λ is

$$\frac{d^2 \Pr\left[\bar{p}(\cdot) \le p^*(\tilde{\zeta}) < \sigma\right]}{d\lambda^2} = b^2 \left(\frac{v^{nf}}{v^{of}}\right)^{-2\delta} e^{-\lambda b \left(\frac{v^{nf}}{v^{of}}\right)^{-\delta}} - a^2 \left(\frac{v^{nf}}{v^{of}}\right)^{-2\delta} e^{-\lambda a \left(\frac{v^{nf}}{v^{of}}\right)^{-\delta}}. \qquad (47)$$

By imposing that this derivative be strictly negative, we obtain

$$\lambda < 2 \left(\frac{v^{nf}}{v^{of}}\right)^{\delta} \frac{\ln(a) - \ln(b)}{a - b} = 2\lambda^*, \qquad (48)$$

a condition that is obviously satisfied for $\lambda = \lambda^*$. Because there is no other λ that would satisfy the first-order condition, λ^* is the global maximum of $\Pr\left[\bar{p}(\cdot) \le p^*(\tilde{\zeta}) < \sigma\right]$ on $(0, \bar{\lambda})$. ∎

Proof of Proposition 6. The comparative-static result for v^{nf}/v^{of} follows immediately from equation (29). With regard to the effects of α^h/β and σ, we note that the expression for λ^* can be rewritten as

$$\lambda^* = \left(\frac{v^{nf}}{v^{of}}\right)^{\delta} \frac{\ln[a(\sigma)] - \ln\left[b\left(\bar{p}\left(\alpha^h/\beta, \sigma\right)\right)\right]}{a(\sigma) - b\left(\bar{p}\left(\alpha^h/\beta, \sigma\right)\right)}, \qquad (49)$$

where a and b are defined in the proof of Proposition 5 and $\bar{p}(\alpha^h/\beta, \sigma)$ is given by equation (16). Using equation (49), we obtain

$$\frac{\partial \lambda^*}{\partial(\alpha^h/\beta)} = \left(\frac{v^{nf}}{v^{of}}\right)^{\delta} \left[\frac{-\frac{1}{b}(a - b) - \ln(a/b)(-1)}{(a - b)^2}\right] \frac{db}{d\bar{p}} \frac{\partial \bar{p}}{\partial(\alpha^h/\beta)}. \qquad (50)$$

Obviously, $(v^{nf}/v^{of})^{\delta}$ and $db/d\bar{p}$ are both strictly positive. From equation (16), it follows that $\partial \bar{p}/\partial(\alpha^h/\beta) < 0$. Finally, $\sigma > \bar{p}$ and hence $a/b > 1$. As a result, the expression in $[\cdot]$ is strictly negative. We therefore conclude that $\partial \lambda^*/\partial(\alpha^h/\beta)$ is strictly positive.

Again using equation (49), one can show that $\partial \lambda^*/\partial \sigma < 0$ is equivalent to

$$\frac{1}{\sigma \bar{p}} \frac{\sigma(1 - \sigma) + \bar{p}(1 - \bar{p})}{(1 - \sigma)^2 + (1 - \bar{p})^2} < \frac{\ln(a/b)}{\sigma - \bar{p}}. \qquad (51)$$

Before continuing with the analysis of condition (51), we note that restriction (R1) implies

$$\sigma > \underline{\sigma} \equiv (1/2)\left(1 + \beta/\alpha^h\right). \qquad (52)$$

Further observe that

$$\lim_{\sigma \to \underline{\sigma}} \bar{p}(\alpha^h/\beta, \sigma) = \underline{\sigma}' \quad \text{and hence} \quad \lim_{\sigma \to \underline{\sigma}} a = \lim_{\sigma \to \underline{\sigma}} b = \frac{\sigma}{1 - \underline{\sigma}}. \tag{53}$$

So both the left-hand side, $LHS(\sigma)$, and the right-hand side, $RHS(\sigma)$, of condition (51) are functions of σ on $(\underline{\sigma}, 1]$. For σ approaching its lower bound, we obtain

$$\lim_{\sigma \to \underline{\sigma}} LHS(\sigma) = \lim_{\sigma \to \underline{\sigma}} RHS(\sigma) = \frac{4}{1 - (\beta/\alpha^h)^2},$$

where one has to use L'Hôpital's rule to derive the limit of $RHS(\sigma)$. To establish that condition (51)—i.e., $LHS(\sigma) < RHS(\sigma)$—holds on $(\underline{\sigma}, 1]$, it is thus sufficient to prove that (i) $LHS(\sigma)$ is a strictly decreasing function on $(\underline{\sigma}, 1]$ and that $RHS(\sigma)$ is strictly greater than $4\left[1 - (\beta/\alpha^h)^2\right]^{-1}$ on $(\underline{\sigma}, 1]$. As can be shown in tedious calculations, both (i) and (ii) hold. ∎

Lemmas

Proof of Lemma 1. The first partial derivative of V with respect to x_{nf} is given by

$$V_{x_{nf}} = \delta v^{nf} \frac{\bar{p}}{1 - \bar{p}} (x_{of})^\delta \lambda \frac{e^{-\lambda \frac{\bar{p}}{1-\bar{p}} \left(\frac{x_{of}}{x_{nf}}\right)^\delta}}{(x_{nf})^{\delta+1}} - 1. \tag{54}$$

Note that $0 < \bar{p} < 1$ (Proposition 2) and $0 < x_{of} < \infty$ (by assumption). As x_{nf} approaches 0, the exponetial function of x_{nf} in equation (54) falls faster towards 0 than does the power function of x_{nf} (see e.g. Sydsæter and Hammond, 2008, p.253). It thus follows that $\lim_{x_{nf} \to 0} V_{x_{nf}} = -1$, implying that V has a boundary local maximum at $x_{nf} = 0$. Turning to the possible interior local maximum, note the following: (i) equation (54) immediately implies $\lim_{x_{nf} \to \infty} V_{x_{nf}} = -1$; (ii) one can show that

$$V_{x_{nf}, x_{nf}} = \begin{cases} > 0 & : & x_{nf} < \bar{x}_{nf} \\ = 0 & : & x_{nf} = \bar{x}_{nf} \\ < 0 & : & x_{nf} > \bar{x}_{nf} \end{cases}, \tag{55}$$

where

$$\bar{x}_{nf} = \left(\lambda \frac{\delta}{1 + \delta} \frac{\bar{p}}{1 - \bar{p}}\right)^{1/\delta} x_{of} > 0. \tag{56}$$

Result (ii) implies that $V_{x_{nf}}$ has a unique maximum on $[0, \infty)$ at $\bar{x}_{nf}$. Given this, we now have to distinguish two cases: $V_{x_{nf}}(\bar{x}_{nf}, \cdot) \leq 0$ and $V_{x_{nf}}(\bar{x}_{nf}, \cdot) > 0$. In the former case, $V_{x_{nf}}$ is non-

positive for all $x_{nf} \in [0, \infty)$, implying that V does not have an interior local maximum. In the latter case, as x_{nf} rises from $\bar{x}_{nf}$ towards ∞, results (i) and (ii) imply that $V_{x_{nf}}$ monotonically falls towards -1, thereby crossing the 0-threshold exactly once. It follows that this intersection is the unique interior local maximum. ■

Proof of Lemma 2. The first partial derivative of W with respect to x_{of} is given by

$$W_{x_{of}} = \delta v^{of} \frac{\bar{p}}{1 - \bar{p}} \frac{1}{(x_{nf})^\delta} \lambda \frac{e^{-\lambda \frac{\bar{p}}{1-\bar{p}} \left(\frac{x_{of}}{x_{nf}} \right)^\delta}}{(x_{of})^{1-\delta}} - 1. \tag{57}$$

Note that $0 < \bar{p} < 1$ (Proposition 2) and $0 < x_{nf} < \infty$ (by assumption). From equation (57), we can infer that $\lim_{x_{of} \to 0} W_{x_{of}} = \infty$ and $\lim_{x_{of} \to \infty} W_{x_{of}} = -1$. Moreover, equation (57) implies that $W_{x_{of}}$ is a strictly decreasing function of x_{of} on $[0, \infty)$. Given these results, the lemma immediately follows. ■