The Economics of Beliefs under Fundamental Uncertainty

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Abstract

Introducing the concept of "belief entrepreneur", this paper offers a novel framework of endogenous belief formation under fundamental uncertainty. We consider a generic setup in which individuals must choose between a tested approach (supplied by a "defender") and a competing innovative approach (supplied by an "innovator"). While the innovation is promising, its true merits are uncertain. Facing an ambiguous choice, individuals turn to heuristic belief formation. As a result, beliefs become contested quantities that arise in a game between the innovator and defender who act as competing belief entrepreneurs. We clarify the conditions under which the contest outcome predominantly reflects information on the merits of the innovation—and when other factors, such as the entrepreneurs' payoffs, dominate. Our analysis may be helpful to policy makers who have to assess uncertain innovations whose spread is propelled by highly favorable beliefs.

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1 Introduction

Societies committed to liberal democracy and capitalism are often viewed as unrivaled "discovery engines". Popper (1945) describes liberal democracy as an incubator for innovative policy ideas, while according to Schumpeter (1942) capitalism is a force that

> *(...) incessantly revolutionizes the economic structure from within, incessantly destroying the old one, incessantly creating a new one.* (p. 83).

Because of these innovation forces, societies committed to liberal democracy and capitalism often encounter novel situations in which data and experiences from the past are of little help in forming beliefs about the consequences of different courses of action. Yet, in practice, decisions have to be taken—and deliberate decisions require some belief. How do people form beliefs in novel situations? It is unlikely that beliefs simply fall "out of the sky", in a completely exogenous fashion. We argue that beliefs emerge endogenously, influenced by familiar economic and psychological forces. This paper offers a simple model based on a contest framework that shows how these forces interact to shape subjective beliefs in novel situations that arise due to innovations.

Many discoveries—technological, policy-related, or institutional—are associated with a move into "uncharted territory": when an innovation emerges, there is often a scarcity of relevant data, making it hard to form an "objective" belief about the innovation's up- and downsides. Innovations thus often come with what we call *fundamental uncertainty*, i.e. with uncertainty (in the sense of Knight 1921) related to their economic fundamentals. As an example, consider the share of jobs that over the next two decades will be lost to automation and artificial intelligence. Existing suggestions for the distribution of possible outcomes vary widely (Segal 2018)—and it is all but impossible to discriminate between them based on existing data. In a similar vein, in the 1990s and early 2000s, there was fundamental uncertainty about the consequences of securitization of US residential mortgage loans. Obviously, there is a long list of fitting examples. At the macro level, it includes the consequences of large-scale quantitative easing by central banks in the aftermaths of the Great Recession and, more recently, the Corona Crisis. At the micro level, ventures into uncharted territory include novel products, production methods, and business models in general—as well as their possible regulation (e.g., "big tech").[1] Section 3 offers a discussion of the securitization example.

A cornerstone of our analysis is that, in the context of any innovation, there are actors with

[1] The examples mentioned here are all among the "many phenomena of interest, [for which] probabilities cannot be claimed to be scientifically measurable quantities." (Gilboa et al. 2014, p. 1409).

a clear incentive to influence the formation of subjective beliefs. Investment bankers may see their incomes rise if the demand for a novel product in the realm of financial engineering is high because market participants have adopted a generally positive belief about the innovation. Similarly, a generally positive belief about a proposal for a novel policy means a higher chance that the proposal will pass the responsible committee or legislature—and so advances the interests of the proposal's sponsor. But often, when there are winners, there are also losers. And those who expect disadvantages from an innovation have an incentive to influence subjective beliefs in a negative way. Thus, the pattern that emerges is one of a competitive game between different *belief entrepreneurs* that try to instill the desired subjective belief into the relevant public. Section 3 exemplifies this generic pattern with the help of a particular innovation, the securitization of US residential mortgage loans in the 1990s.

But why do clearly self-interested actors obtain a chance to influence beliefs? Innovations involve fundamental uncertainty. It is often impossible to obtain an objective estimate $\hat{p}$ for a probability p that at time t innovation I produces outcome O—with the result that I cannot be valued objectively. Similarly, individuals are hardly in a position to immediately come up with a firm *subjective belief* about p; they need to make up their minds first. So, when an innovation emerges, individuals a priori see themselves confronted with an ambiguous valuation and decision problem.[2] Yet, as we know from a large literature (Ellsberg 1961, Machina and Siniscalchi 2014), individuals dislike ambiguity and show a tendency to avoid ambiguous options. But do people generally sacrifice innovation just because of ambiguity? We think not. The human mind has an alternative approach: the literature on heuristics (Kahneman and Tversky 1973; Tversky and Kahneman 1974; Slovic et al. 2007; Gennaioli and Shleifer 2010) shows that decision makers are prone to substituting difficult problems with seemingly analogous, yet simpler ones. We suggest that an ambiguous valuation and decision problem relating to an innovation is perceived as difficult—and therefore replaced by what appears to be an analogous, but non-ambiguous problem. This mechanism of *heuristic substitution* offers a measure of influence to belief entrepreneurs: they are competing suppliers of heuristic analogies that aim to push subjective beliefs into a desired direction.

The mechanism of heuristic substitution is consistent with recent research in neuroscience, in particular with predictive coding theory (Friston 2010; Clark 2013, 2016; Barrett 2017b) which understands the brain as a "prediction machinery" (in the sense of statistical learning) that uses past experiences as "training data". Importantly, this data need not coincide with what would

[2] Fundamental uncertainty or uncertainty in the sense of Knight (1921) means that event probabilities cannot be objectively established. Our use of "ambiguous" is based on Ellsberg (1961): a decision problem is ambiguous if the agent has little confidence in their estimates of probabilities (also see Machina and Siniscalchi 2014).

count as relevant data under standard economic definitions of rationality. According to the theory, when encountering an unfamiliar instance (e.g., an innovation), the brain proactively employs analogies, associations, and mental simulations to generate predictions about the valuation of that instance based on its internal models (Bar 2007; Barsalou 2009).

Our model considers a generic business or political domain in which there is an incumbent *defender* of the status quo. The defender is experienced in the use of a tested *approach* for producing an outcome. An approach can mean different things: a production method or business model, an economic policy, or an institution. We consider the domain in a particular moment: an *innovator* has emerged and now proposes an innovative approach for producing the outcome in question. Being novel, the innovative approach is subject to fundamental uncertainty: while it is clear that the innovation is superior in one state of the world and inferior in the other, the probabilities of the states are unknown. Yet objective information on the state need not be entirely absent: the defender and the innovator receive a signal whose informativeness, a key parameter of the analysis, can range from zero to perfect. The signal is private and cannot be credibly conveyed to third parties. A continued use of the tested approach, *irrespective* of the state of the world, implies an immediate gain for the defender (e.g., in the form of short-run profits in the case of a business domain), while a broad adoption of the innovative approach leads to an immediate gain for the innovator.

Besides the defender and the innovator, there is a *target audience*, a group of individuals who decide which one of the two approaches to use. The audience's objective is to choose the superior approach, either individually or collectively. Audience members are averse to ambiguity (caused by fundamental uncertainty) and thus receptive for ambiguity-suppressing subjective beliefs. And it is precisely this receptiveness that turns the members' individual beliefs about the merits of the innovation into contested magnitudes, i.e., into endogenous variables that are determined in a competitive game between the innovator and the defender. We capture this competition by means of a *belief contest* whose coveted prize is the broad adoption of the desired subjective belief (either positive or negative towards the innovation). For the belief entrepreneurs, taking part in a belief contest means spending resources—creativity, time, money—on activities that in practice include devising and disseminating suitable narratives (that either support or oppose the innovation). In addition to these short-run expenses, acting as a belief entrepreneur may entail a "long-run" cost: promoting an approach that later is revealed as inferior causes reputational damages.[3]

[3]To keep the exposition concise, the main part of the paper focuses on a variant of the model in which the members of the target audience decide individually on which approach to use. A sketch of a model variant with collective choice (committee decisions or decisions by the public at large) can be found in Appendix A.

Our analysis identifies three factors that influence the set of subjective beliefs the members of the target audience eventually adopt: the information contained in the signal to the belief entrepreneurs ("information"); the magnitudes of the gains and losses (both immediate and long-run) that may come with the participation in the belief contest ("payoffs"); and accidental forces, such as a narrative suddenly going viral ("luck"). Yet only the first factor is related to the actual state of the world, i.e., to the sole "fundamental" determining whether the innovative approach is superior or inferior. We clarify how much influence each of these three factors commands under different circumstances. A key magnitude is the ratio of possible immediate gains by the innovator to those by the defender, with equilibrium beliefs growing more innovation-friendly as the ratio rises. Holding constant the informativeness of the signal, the influence of the immediate gains ratio is stronger if the two belief entrepreneurs discount possible long-run reputational damages more heavily.

A key prediction of our model is that, no matter the information conveyed by the signal, equilibrium beliefs are highly innovation-friendly if the immediate gains anticipated by the innovator are comparatively large and potential long-run reputational damages are heavily discounted. Put differently, in this constellation, there is an absence of correlation between the information conveyed by the signal and the equilibrium beliefs held by the target audience. Not so in a second constellation: if the ratio of anticipated immediate gains by the innovator and defender is balanced and possible reputational damages loom large, the model predicts a positive correlation between the information conveyed by the signal and equilibrium beliefs. The securitization of US residential mortgages discussed in Section 3 is a showcase example of the first constellation: if successfully introduced to financial investors, securitization would generate large short-term profits due to the enormous demand for "safe" assets; moreover, since a meaningful test of the innovation would require a rare housing bust, the window of uncertainty was long, implying that potential reputational damages could be discounted heavily. The contest turned out as our model would have predicted: many financial investors developed highly favorable beliefs towards the innovation. In this, luck in the form of the then Fed chairman adopting the innovators' narratives certainly played a helpful role, too. When the housing bust finally arrived, the favorable beliefs were shattered.[4]

The above discussion of constellations illustrates that our framework can serve as a diagnostic tool for policy makers that have to grapple with an innovation whose fundamentals they do not know any better than the public at large. In particular, the framework suggests what

[4] Goldfarb and Kirsch (2019), who focus on uncertainty and major innovations, offer a number of candidates for the second constellation. One of them (emerging roughly at the same time as securitization) is LCD displays.

data the policy makers should collect and analyze in such a situation. The list includes data on anticipated short-term profits as well as information that allows them to assess the current weight of potential reputational damages (length of the window of uncertainty, time horizon of decision makers). If, for instance, the data is suggestive of the constellation expemplfied by securitization, the positive belief that propels the innovation cannot be considered informative about the fundamental; so the policy makers have a particularly strong reason to prepare for the eventuality that the innovation will prove harmful later on.

The rest of this paper is organized as follows. The next section discusses the related literature. Section 3 provides a concrete example of the basic constellation that is captured by our model of belief formation. Section 4 presents the model. In Section 5, we derive the outcome of the belief formation process in a simplified setting in which the campaigning efforts chosen by the belief entrepreneurs are binary variables; the section also includes a discussion of a number of key insights. The analysis of the more general setup, along with a discussion of additional insights, is contained in Section 6. Finally, Section 7 concludes.

2 Related Literature

In our setting, the principal actors are belief entrepreneurs who compete in shaping beliefs about an innovation by supplying heuristic analogies. Mokyr (2013, 2017), analyzing the coevolution of culture and capitalism, develops the closely related concept of "cultural entrepreneur". Mokyr defines culture as a "system of beliefs, values and preferences that shape [...] institutions." Cultural entrepreneurs are understood as "individuals who refuse to take the existing technology or market structure as given and try to change it." (Mokyr 2013, p. 2). Their essential role is to provide a perspective on new social orders and on how nature can be analyzed and harnessed for the development of new technologies. While the concept of belief entrepreneur links our analysis with Mokyr's work, there are several important differences. First, in our setup, belief entrepreneurs may be driven by "material" motives; rather than considering intellectuals, we focus on business people, politicians, and technocrats (i.e., on actors that more commonly populate economic models). Second, we investigate a contest between belief entrepreneurs with opposing interests; our hypothesis is that innovations create losers who have strong incentives to fight the broad adoption of a particular innovation. Third, the analysis here relies on formal modeling.

Bénabou et al. (2020) put forward the idea of a "narrative entrepreneur" in their analysis of narratives and moral behavior. They investigate the conditions under which individuals use narratives as exculpatory or responsibilizing arguments to either justify selfish behavior or encourage pro-social behavior that is individually costly. In an extension, Bénabou et al. (2020) allow agents to actively search for such narratives and thus become "narrative entrepreneurs". In Bénabou et al. (2020), agents assume one of two possible unobserved types that assign either a low or high value to a moral action; both types share an equal concern for being perceived as morally responsible. Searching and sharing of narratives thus serves for strategic signaling about their types. In the setting of Bénabou et al. (2020), there is no competition between different narrative entrepreneurs. In our analysis, belief entrepreneurs aim to influence an audience's belief about the merits of an innovation that is beneficial to the respective belief entrepreneur's business (or otherwise). We do not distinguish between different unobserved types of entrepreneurs; rather, our focus is on how their competition affects prior beliefs about the innovation as an equilibrium outcome.

This paper rests on the idea that individuals deal with a difficult problem—the assessment of an innovation under fundamental uncertainty—by substituting it with a simpler one that features an analogy to the original problem. Narratives are typical carriers of such analogies. Shiller (2017, 2019) provides an extensive treatment of narratives and offers many examples of narratives that influenced beliefs about novel ideas by business people or politicians. Narratives may affect belief formation via the representativeness heuristic by influencing what comes to mind in the context of a valuation problem that is subject to uncertainty and ambiguity. Gennaioli and Shleifer (2010) offer an explicit model of this "what-comes-to-mind" mechanism and show how the mechanism can explain many otherwise puzzling judgment biases such as the conjunction and disjunction fallacies. Our analysis does not directly model any specific heuristic but rather capture their use—in the context of narratives—in a reduced-form way. Reasons for the prevalence and effectiveness of narratives have recently been considered from a neuroscience and evolutionary perspective (Armstrong 2020). Humans are a uniquely cooperative species (Fehr and Gächter 2002). An important question concerns how such cooperation is achieved. Key answers are altruistic punishment (Fehr and Gächter 2002; Fehr and Fischbacher 2003) and also that trust (towards members of an in-group) tends to "feel good" (Fehr et al. 2005). However, besides altruistic punishment and trust, narratives, too, are thought to have the potential to support (within-group) cooperation. They may do so by fostering "shared intentionality" (Tomasello and Rakoczky 2003; Armstrong 2020). This view is supported by the fact that narratives have a unique potential to turn viral—a major theme in Shiller (2019)—and thus can be highly effective in promoting a shared goal at the group level. In turn, this may explain why humans are predisposed to "listen" to narratives. In our model, belief entrepreneurs take advantage of this predisposition.

We finally refer to some of our previous work. In Binswanger and Oechslin (2015), we consider a setting with fundamental uncertainty in which different societal groups hold dissenting subjective beliefs about the success probability of an economic reform. We show that fundamental uncertainty can lead to political gridlock and stagnation. But in this previous analysis, subjective beliefs are regarded as exogenously given.

3 Beneficial Innovation or "Hydrogen Bomb"?

This section discusses a concrete example of the basic constellation captured by our model. We consider securitization, a past innovation in the financial industry. The term securitization refers to a business model that in the late 1980s started to emerge in the domain of US commercial banking. Securitization promised a novel approach to dealing with the credit risks inextricably linked to the lending business—an approach that *potentially* would lead to a more efficient outcome in terms of the economy-wide risk allocation and so improve financial stability. For a bank, the tested approach was to manage a diversified portfolio of loans that it retained on its balance sheet. The innovative approach, securitization, consisted in distributing those loans, including large parts of the associated credit risk, to third-party investors. Specifically, a bank would pool a certain number of loans and then re-finance them through issuing securities—collateralized debt obligations—with different levels of "riskiness" (from super-senior to junior); the bank would then sell those securities to third-party investors with different levels of risk capacity. The latter, in turn, could redistribute the risks once again using an accompanying innovation called credit derivatives (such as credit default swaps).

While securitization and credit derivatives were successfully applied in some segments of the credit market, it was fundamentally uncertain whether these innovations would improve the risk allocation in other segments, too. For instance, when it came to the securitization of residential mortgage loans, a scarcity of data bedeviled the estimation of the probabilities with which the resulting securities would default. A particular problem concerned the correlations of defaults among pooled mortgages. Home prices had been growing more or less continuously since World War II (Gennaioli et al. 2012). So there was uncertainty regarding the extent to which the historical data would be meaningful in the event of a large common shock to the housing market (Dungey et al. 2013). Industry insiders spotted signs of the severity of the problem early on. Evidence of this is that J.P. Morgan, a pioneer of securitization, abandoned the business of packaging mortgages as early as in the late 1990s (Flood 2009).[5] But many others remained unperturbed and the market—driven by a large demand for "safe" assets (Gennaioli et al. 2012)—virtually exploded in the following years.

Referring to securitization in general, the role of the incumbent defender was assumed by the "community" of senior commercial bankers with a large stake (e.g., in terms of compensation and career perspectives) in the continuing dominance of traditional balance sheet lending. The part of the innovator, on the other hand, was played by the "community" of members of banks' securitization and credit derivatives teams. Vying for the very same compensations and career perspectives, the latter wanted to scale up the innovative approach, i.e., turn it from an obscure method used in some segments of the credit market into the standard way of dealing with credit risk. The target audience consisted of people inside and outside commercial banking, including bank CEOs and board members (who would determine the scope for securitization within their institutions) as well as regulators, rating agencies, and professional investors (whose decisions would critically affect the profitability of the innovative approach).

Both communities were engaged in a contest whose prize was the wide adoption of the desired ambiguity-suppressing subjective belief about securitization by the target audience. The community of innovators supplied supporting narratives, while the community of defenders pushed opposing ones. For concreteness, consider narratives (primarily) aimed at the regulators. Supporting narratives were designed to nurture optimistic beliefs by promoting a particular substitution: when evaluating securitization and credit derivatives, regulators should substitute the complex and imperfect real-world financial system with something close to the well-understood and orderly perfect-markets benchmark. In that benchmark, market discipline—with its self-regulating forces—would limit the demand for (and sale of) securities that are ill-priced due to misjudged or unquantifiable risks. And in doing so, the market would minimize any threat to financial stability. Tett (2009) quotes an influential member of the community of innovators (Mark Brickell, a J.P. Morgan banker and key contributor to a 1993 report on credit derivatives by the Group of Thirty) as remarking:[6]

> *Markets can correct excesses far better than any government. Market discipline is the best form of discipline there is.* (p. 36).

From today's perspective, the boldness of this statement may surprise. Yet it comes from the

[5]Later on, such data uncertainties started to attract broader attention. For instance, in 2005, the Bank for International Settlements' Committee on the Global Financial System (BIS 2005, p. 24) warned: "As a result [of uncertain default correlations], model-based risk assessments can be a long way from 'true' values (...)."

[6]The G30 report was a compendium written by banking industry representatives. It contained, among other things, advice and "best practice" norms and was an attempt to forestall regulation by the government.

early 1990s, a time when governments were often the cause of economic illnesses, while "the market" was considered a miracle cure. Regulators should transfer this logic to the new field of financial engineering. And they did. Brickell's opinion was echoed by Alan Greenspan, then the chairman of the Federal Reserve. In a speech on May 25, 1994, the Fed chairman (according to a transcript dated July 1) testified to selected members of the House of Representatives that:

> *Legislation directed at [credit] derivatives (...), absent broader reform, could actually increase risks in the US financial system by creating a regulatory regime that is itself ineffective and that diminishes the effectiveness of market discipline.*

Opposing narratives, on the other hand, were designed to nurture pessimistic beliefs by promoting a different kind of substitution. According to those narratives, the real-world financial system would have little in common with the perfect-markets benchmark and thus would not curb the sale of ill-priced securities and credit derivatives. Over time, ill-priced securities and derivatives would do to the financial system what weapons of mass destruction do to the enemy in a war. According to Tett (2009), Felix Rohatyn—a "legendary Wall Street figure" and advocate of the tested approach to dealing with credit risk—called

> *(...) [credit] derivatives financial hydrogen bombs, built on personal computers by twenty-six-year olds with MBAs.* (p. 36).

In a 2003 letter to shareholders (dated February 21), Warren Buffett—Chairman of Berkshire Hathaway, a company that was among other things a large provider of more traditional financial services—took a similar position by saying that credit derivatives were

> *(...) financial weapons of mass destruction, carrying dangers that, while now latent, are potentially lethal.* (p. 15).

In retrospect, it is fair to say that the belief contest was won by the innovators. Apparently, their narratives struck a chord with regulators and other influential members of the target audience. Regulation remained light-touch and the securitization of, most notably, residential mortgage loans became a thriving business. But when in 2007 US house prices started to slide across the board, this mortgage-related business was put to a serious test—and it failed. The early doubts about the default correlations were revealed to be well founded. Instead of improving the risk allocation, the novel approach had mostly masked the credit risks. As a result, the involved banks suffered a heavy reputational loss and have paid hundreds of billions of dollars in fines since the end of the financial crisis (e.g., BCG 2017).

4 The Model

4.1 Basic Setup

There are two possible approaches $a \in \{0, 1\}$ for producing an outcome: a tested approach ($a = 0$) and an innovative approach ($a = 1$). The tested approach leads to a known outcome $\rho > 0$. The innovative approach produces an unknown outcome θ. There are two states of the world, $F \in \{H, L\}$. In state H, the outcome of the innovative approach is $\bar{\theta} > \rho$, while in state L the outcome is 0. In the beginning, state F is not observed and subject to fundamental uncertainty in the sense of Knight (1921). In particular, there is a lack of data that would suggest any specific objectively-valid belief about $\Pr[F = H]$. However, if the demand for the innovative approach turns out to be positive, state F will be revealed eventually. In what follows, we will often refer to F as the innovation's "fundamental".

Decisions on the use of the two approaches are taken by the members of a "target audience" (which is of unit size). Yet not only is there no basis for an objective belief about $\Pr[F = H]$; in addition, audience members are not in a position to immediately come up with a subjective belief. Rather, they find themselves confronted with an ambiguous valuation and decision problem. Subsection 4.2 addresses how audience members cope with ambiguity. We consider two alternative types of decision making, *individual* choice and *collective* choice. In the main part of the paper, the focus is on individual choice: each audience member decides for themselves which one of the two approaches to use. Appendix A assumes collective choice.

The two approaches are offered by competing suppliers, the innovator (I) and the defender (D). Obviously, the former provides the innovative approach, while the defender is the established supplier of the tested approach—and potentially faces displacement by the innovator. As we will discuss below, each has an incentive to influence the process of belief formation by the members of the audience. As a result, we will call them belief entrepreneurs.

4.2 Belief Formation and Belief Contest

Members of the target audience need to choose between a tested approach with a known outcome and an innovative approach with an ambiguous outcome. So, with respect to the innovation, members of the audience a priori see themselves confronted with an ambiguous valuation and decision problem—or with an instance of "experiential blindness" (Barrett 2017a). What is a realistic behavioral assumption for these circumstances? We find it natural to draw on the psychological literature on heuristics (Kahneman and Tversky 1973; Tversky and Kahneman 1974; Slovic et al. 2007; Gennaioli and Shleifer 2010). The robust pattern found in this literature is that, when decision makers face a difficult problem, they heuristically substitute it with a seemingly analogous, yet simpler, one—and then adopt the solution to the latter. Our premise is that an ambiguous valuation and decision problem relating to an innovation is perceived as difficult; accordingly, it will be replaced by what appears to be an analogous, but non-ambiguous, valuation and decision problem. The relevance of this type of heuristic substitution is supported by research in neuroscience (Bar 2007; Barsalou 2009).

In what follows, we assume that the members of the audience embrace heuristic substitution, a behavior that offers a measure of influence to the innovator and the defender. The innovator may want to put an effort into making audience members adopt a substitute valuation problem that results in a positive subjective belief towards the innovation. In a similar vein, the defender has an incentive to entice audience members into adopting a substitute valuation problem that leads to a negative belief. Put differently, heuristic substitution turns the members' subjective beliefs into contested magnitudes, i.e., into endogenous variables that are determined in a competition between the two belief entrepreneurs. We capture this competition by means of a contest game, a widely used modeling device for situations in which two or more parties compete for the same prize.[7] In our case, the prize to be won by one of the two belief entrepreneurs is a generally positive attitude towards their own approach. Since audience members select just one of the competing approaches, a generally positive belief towards the innovation means a win for the innovator and a loss for the defender (and vice versa).

We denote by $\Pr_i[F = H]$ the subjective individual belief adopted by audience member i about the chance that the fundamental of the innovation is sound. In this regard, we assume

$$\Pr_i[F = H] = \alpha_i \pi, \tag{1}$$

where $\alpha_i \in [0, \bar{\alpha}]$ is an individual-specific parameter that is distributed according to a distribution function $H(\alpha)$ and π is the component of individual subjective beliefs that is shared by all audience members. In what follows, we call π the belief anchor. If π takes a high value, the attitude towards the innovation is generally positive; if π takes a low value, it is generally negative. Actual individual subjective beliefs deviate from the anchor in both directions. If $\alpha_i > 1$, audience member i holds a more positive belief about the innovation (compared to the anchor); if $\alpha_i < 1$, the belief is more negative. The α_is reflect individual personality traits. For instance, a high α_i describes an individual that is "open to new experiences" or that shows

[7] Standard applications include competitions for access to a natural resource and competitions for large vote or market shares (see Konrad 2009 and Corchòn and Marini 2018). Falkinger (2007) develops an alternative concept of a "contest for attention" that is inspired by empirical findings on the psychology of attention.

little "risk aversion" (in a broad sense of the term). These characteristics can be innate or acquired through upbringing and personal experiences (see, e.g., McAdams and Pals 2006 for an illuminating discussion on personality characteristics).

The belief anchor π is determined according to a usual contest success function of the form

$$\pi = \begin{cases} 0 & \text{if } e_I = e_D = 0 \\ \frac{1}{\bar{\alpha}} \frac{\zeta e_I^\eta}{\zeta e_I^\eta + e_D^\eta} & \text{otherwise} \end{cases}, \tag{2}$$

where e_I and e_D refer to the effort levels by the innovator and the defender, respectively, and $0 < \eta < 1$ captures the decisiveness of the contest (i.e., how fast differences in effort levels translate into outcome changes). ζ stands for a non-negative random variable whose density function $\psi(\zeta)$ is known to the belief entrepreneurs. As will become clear below, random variable ζ is linked to the way the two belief entrepreneurs fight their contest. Finally, dividing by $\bar{\alpha}$ ensures that all subjective beliefs fall in the range $[0, 1]$.

Contest success function (2) implies that π takes a large value if the innovator's effort is high *relative* to the defender's. However, in practice, how would belief entrepreneurs spend their resources? Devising and disseminating narratives is certainly part of the answer.[8] Narratives are instrumental in the substitution process as they are the carriers of analogies. Returning to the securitization example of Section 3, an analogy put forth by the innovators was a theoretical benchmark: the audience should evaluate securitization through the lens of the well-understood and orderly perfect-markets benchmark rather than against the backdrop of imperfect and complex real-world financial markets; if this substitution were embraced, securitization would be seen as strengthening Adam Smith's "invisible hand".

Shiller (2019) offers many more examples of how narratives can "help" in the process of substitution. Moreover, he points out that narratives often include an emotional component that makes them contagious. However, potency and contagiousness of a particular narrative are subject to substantial randomness and cannot be perfectly controlled. Many examples suggest that a seemingly minor detail can decide whether a narrative "goes viral" or not. As a result, a competing belief entrepreneur can hardly ensure that their own narrative is superior to those pushed by opponents—even if the latter are vastly outdone in terms of resources that go into the design and dissemination of a narrative.[9] Introducing ζ into contest success function

[8]For instance, it takes time and money to devise and test narratives in a systematic way. Similarly, enlisting opinion leaders, journalists, or celebrities in an effort to spread a narrative requires resources.

[9]Comparing narratives to movies, Shiller (2019, p. 41) quotes a former president of the Motion Picture Association of America: "With all the experience, with all the creative instincts of the wisest people in our business, *no one, absolutely no one* can tell you what a movie is going to do in the marketplace...".

(2) captures that the potency of a narrative depends on accidental forces that are beyond the innovator's control. Obviously, the same holds for the defender's narratives relating to the innovation. Yet introducing a random variable that multiplies e_D would serve no purpose: it would always only be the ratio of the two random variables that matters. So we assume that only one of the effort levels is affected by a random force.

Our assumptions so far imply that audience member i, when considering the two available approaches, (subjectively) expects the following outcome:

$$E_i\{U(a_i)\} = \begin{cases} \rho & \text{if } a_i = 0 \\ \alpha_i \pi \, \bar{\theta} & \text{if } a_i = 1 \end{cases}. \tag{3}$$

The second line in equation (3) reflects that $\theta = 0$ if $F = L$. In what follows, we refer to equation (3) as audience member i's objective function.

4.3 Belief Entrepreneurs' Objective Functions

By choosing e_I and e_D, the belief entrepreneurs influence the distribution of beliefs in the target audience. In doing so, they affect the levels of *demand* for the innovative approach, D_I, and the tested approach, D_D. As the size of the audience is normalized to 1, we have $D_D = 1 - D_I$, where D_I is the measure of members with $a_i = 1$. Clearly, $D_I(\pi)$ is an increasing function of π, while $D_D(\pi)$ is a decreasing function of π. The total payoff incurred by a belief entrepreneur consists of several elements. First consider the innovator. We use $v_I \geq 0$ to denote the net value of supplying one unit of the innovative approach that materializes *before* state F is revealed. With a slight abuse of language, v_I is called the innovator's *before*-payoff. The parameter w_I^H denotes the additional value that materializes *after* F has been revealed to be high, while $w_I^L \geq 0$ is a cost that is incurred *after* F has been revealed to be low. We refer to these parameters as *after*-payoffs.[10] The innovator holds a subjective belief about $\Pr[F = H]$, called ϕ_I. Given this, we state the innovator's objective function as

$$\begin{aligned} V_I &= E\{D_I(\pi)\} v_I + \delta_I E\{D_I(\pi)\} \left[\phi_I w_I^H - (1-\phi_I) w_I^L\right] - c_I e_I \\ &\equiv E\{D_I(\pi)\} \tilde{v}_I - c_I e_I, \end{aligned} \tag{4}$$

where $\tilde{v}_I \equiv v_I + \delta_I \left[\phi_I w_I^H - (1-\phi_I) w_I^L\right]$. In equation (4), $E\{\cdot\}$ denotes the expectations operator, reflecting that π is influenced by ζ, a random variable whose distribution the belief

[10] We do not use the terms "short-run" or "long-run" payoffs. As it potentially takes a long time before the fundamental is revealed, the term "short-run" could actually be misleading.

entrepreneurs know. The term containing the after-payoffs is weighted by a discount factor, δ_I, as uncovering the fundamental takes time (and there may be turnover in position of decision maker). We assume that the after-payoffs are proportional to the demand for the innovative approach: having campaigned for an innovation that works (does not work) leads to a larger reputational gain (damage) if the innovation has been used more widely. Finally, campaigning costs are captured by $c_I e_I$, where $c_I > 0$ is the cost per unit of effort.

Now consider the defender, whose objective function precisely mirrors that of the innovator:

$$\begin{aligned} V_D &= E\left\{D_D(\pi)\right\} v_D + \delta_D E\left\{D_D(\pi)\right\}\left[(1-\phi_D)w_D^L - \phi_D w_D^H\right] - c_D e_D \\ &\equiv E\left\{D_D(\pi)\right\} \tilde{v}_D - c_D e_D, \end{aligned} \tag{5}$$

with $\tilde{v}_D \equiv v_D + \delta_D\left[(1-\phi_D)w_D^L - \phi_D w_D^H\right]$. Note that for the defender the state being revealed as low means a gain (since the innovation failed). Therefore, in equation (5), $w_D^H \geq 0$ enters with a negative sign, while $w_D^L \geq 0$ enters with a positive one.

Before the game starts, the belief entrepreneurs hold their own prior beliefs about $\Pr[F = H]$. We denote those priors, which are purely subjective, by ϕ_k^0, $k = D, I$.[11] At the start of the game, the belief entrepreneurs receive an identical signal $S \in \{H, L\}$ about the realization of F. The quality of the signal is given by $\sigma \equiv Pr[F = f|S = f]$, $f = H, L$, where σ ranges from a minimum of $1/2$ (no information about fundamental) to a maximum of 1 (no fundamental uncertainty). The signal cannot be credibly communicated to the target audience since its interpretation requires expertise that the audience lacks. Assuming an identical signal is not crucial: we could assume the belief entrepreneurs to receive individual signals that are merely correlated (and not necessarily identical) without changing the behavioral implications of the model. However, working with an identical signal allows us to parameterize fundamental uncertainty in a parsimonious way (with $\sigma = 1/2$ and $\sigma = 1$ delimiting the entire spectrum). Having observed S, the two entrepreneurs update their subjective priors using Bayes' rule. The resulting posterior (subjective) beliefs are given by ϕ_I and ϕ_D (as they appear in equations 4 and 5). In several parts of the analysis, we will directly start with the posteriors, while in other parts we will investigate the role of the signal's quality.

4.4 Timeline

The timing of the game is as follows:

[11] Arguably, without an optimistic prior belief, a potential innovator would hardly put effort into the development of a novel approach. So, in practice, ϕ_I^0 is likely to take a high value. See Herz et al. (2014) for experimental evidence on the relationship between entrepreneurial optimism and innovation.

1. Nature determines $F \in \{L, H\}$, the fundamental of the innovation; the two belief entrepreneurs draw their prior beliefs, ϕ_I^0 and ϕ_D^0.
2. Nature sends a signal $S \in \{L, H\}$ about F to the belief entrepreneurs; observing S, the belief entrepreneurs form their posterior beliefs, ϕ_I and ϕ_D.
3. The belief entrepreneurs simultaneously choose the levels of their campaign efforts, e_I and e_D; nature draws ζ; the belief anchor, π, forms.
4. Each member i of the target audience adopts a subjective belief about $\Pr[F = H]$ and then decides on the approach, $a_i \in \{0, 1\}$; the before-payoffs materialize.
5. If the innovative approach finds users, nature reveals the fundamental of the innovation; the after-payoffs materialize.

The essential stages are 3 and 4 (there are no strategic elements in 1, 2, and 5). Moreover, the analysis of stage 4 is straightforward and can be integrated into that of 3 by means of a simple application of backward induction. De facto, solving the model means solving a static contest game with two contestants, the belief entrepreneurs.

5 Belief Formation in a Simplified Setup

The purpose of this section is to provide a simple and transparent illustration of the principal forces and factors shaping subjective beliefs under fundamental uncertainty. All of the key insights developed here will also hold in the general setup that is studied in Section 6. The general setup, however, will provide some additional insights.

This section makes two simplifying assumptions. First, the belief entrepreneurs must pick one of just two levels of campaigning effort: $e_k \in \{0, 1\}$, $k = I, D$. Second, in contest success function (2), the random factor takes one of only two possible values: $\zeta \in \{0, \bar{\zeta}\}$, where $\bar{\zeta} > 0$; the probability of $\zeta = \bar{\zeta}$ is strictly positive and denoted by ψ.

5.1 Analysis

Given these simplifying assumptions, the outcome of the belief contest is described as follows:

$$\pi(e_I, e_D, \zeta) = \begin{cases} 0 & \text{if } \; e_I = 0; \; \text{ or if } \; \zeta = 0 \\ \frac{1}{\bar{\alpha}} & \text{if } \; e_I = 1, \, e_D = 0, \, \zeta = \bar{\zeta} \\ \frac{1}{\bar{\alpha}} \frac{\bar{\zeta}}{1+\bar{\zeta}} & \text{if } \; e_I = 1, \, e_D = 1, \, \zeta = \bar{\zeta} \end{cases} . \tag{6}$$

The assumptions regarding ρ, $\bar{\theta}$, and $H(\alpha)$ are implicit: we assume a constellation that leads to the following "demand function" for the innovative approach:[12]

$$D_I(\pi) = \begin{cases} 0 & \text{if} \quad \pi = 0 \\ \lambda & \text{if} \quad \pi = \frac{1}{\bar{\alpha}} \frac{\bar{\zeta}}{1+\zeta} \\ 1 & \text{if} \quad \pi = \frac{1}{\bar{\alpha}} \end{cases}, \tag{7}$$

where $0 < \lambda < 1$. So, if the belief anchor takes one of the polar values, the winner serves the entire audience; if π takes the interior value, the audience is split.

Together, equations (4), (5), (6) and (7) imply the payoffs shown in Table 1. To understand the entries, note that the innovator's choice of campaigning effort, e_I, affects the belief anchor only if $\zeta = \bar{\zeta}$ (which happens with probability ψ). As a result, even if the innovator campaigns, the defender still serves the entire audience if $\zeta = 0$. Moreover, observe that, if the outcome is such that the innovative approach is not used at all, F is never revealed and reputational effects are absent. Thus, in those cases, the after-payoffs are identical to zero, implying $\tilde{v}_k = v_k$. This is why some entries in Table 1 show a v_D instead of a $\tilde{v}_D$.

We are interested in what type of Nash equilibrium arises under which parameter constellation. For this discussion, we denote strategy profiles as (e_I, e_D). For instance, $(1, 0)$ means that the innovator invests, while the defender does not. It is obvious that profile $(0, 1)$ cannot be a Nash equilibrium. If the innovator does not campaign, the defender does not want to do so either: as the incumbent, the defender anyway serves the entire audience if the innovator is inactive. However, all other strategy profiles can be equilibria under some parameter constellations. Concerning $\tilde{v}_k$ and c_k we find that it is always only the ratio of the two that matters for the equilibrium outcome. For brevity, we refer to $\tilde{v}_k/c_k$, $k = I, D$, as the *normalized payoff*. Note that $\tilde{v}_k$ combines before-payoffs and after-payoffs.[13]

Table 1 implies that profile $(0, 0)$ is an equilibrium if and only if $\tilde{v}_I/c_I \leq 1/\psi$, i.e., if and only if the innovator's normalized payoff is small (also see Figure 1). Profile $(1, 0)$ is an equilibrium if and only if $\tilde{v}_I/c_I \geq 1/\psi$ and $\tilde{v}_D/c_D \leq 1/(\psi(1-\lambda))$, i.e., in a situation in which the innovator's normalized payoff is large and the defender's is relatively small. The remaining pure-strategy profile $(1, 1)$ is an equilibrium if and only if $\tilde{v}_I/c_I \geq 1/(\psi\lambda)$ and $\tilde{v}_D/c_D \geq 1/(\psi(1-\lambda))$, i.e., if both entrepreneurs stand to win large normalized payoffs.

[12] The parameters ρ and $\bar{\theta}$ appear in objective function (3), on which individual decisions on the use of the approach are based. The distribution function $H(\alpha)$ determines how individual decisions translate into total demand. "Demand function" (7) requires that there be no mass point at $\alpha = 0$.

[13] More precisely, $\tilde{v}_k/c_k$ is the sum of (discounted) payoffs, as expected by belief entrepreneur k, that come with supplying one unit of k's approach, normalized by k's per-unit cost of effort.

		Defender	
		$e_D = 0$	$e_D = 1$
Innovator	$e_I = 0$	0 v_D	0 $v_D - c_D$
	$e_I = 1$	$\psi\tilde{v}_I - c_I$ $(1-\psi)v_D$	$\psi\lambda\tilde{v}_I - c_I$ $(1-\psi)v_D + \psi(1-\lambda)\tilde{v}_D - c_D$

Table 1: Game matrix for belief contest

Under the constellation $1/\psi < \tilde{v}_I/c_I < 1/(\psi\lambda)$ and $\tilde{v}_D/c_D > 1/(\psi(1-\lambda))$, a Nash equilibrium in pure strategies does not exists. The innovator's normalized payoff is too large for profile $(0,0)$ to be an equilibrium. Profile $(1,0)$ cannot be an equilibrium because the defender's normalized payoff is too large for inaction if the innovator campaigns. But what about $(1,1)$? This profile cannot be an equilibrium either because the innovator's payoff is just not large enough to justify a fight against the defender. Since profile $(0,1)$ can never be an equilibrium, it follows that there is no pure-strategy equilibrium at all. But because every game with a finite number of players and profiles has at least one Nash equilibrium, there must be an equilibrium in mixed strategies. Denote by p the probability of the innovator choosing $e_I = 1$ and by q the probability of the defender choosing $e_D = 1$. Then:

$$p = \frac{1}{\psi(1-\lambda)\,\tilde{v}_D/c_D} \qquad \text{and} \qquad q = \frac{\psi - 1/(\tilde{v}_I/c_I)}{(1-\lambda)\psi}. \tag{8}$$

Again, it is normalized payoffs that matter. All else equal, a stronger defender (i.e., one with a larger normalized payoff) makes the innovator *less* likely to enter a belief campaign, while a stronger innovator means that the defender is *more* likely to campaign. Figure 1 summarizes what type of equilibrium arises under which parameter constellation. The x-axis and the y-axis measure the normalized payoffs that, respectively, the innovator and the defender stand to win.

5.2 Discussion

The above simplified setup conveys many of the insights of the more general setup in Section 6. It assembles a set of factors influencing subjective beliefs under fundamental uncertainty and clarifies how they interact. It offers guidance even in the limiting case of a complete absence of objective information on $\Pr[F = H]$ and demonstrates that economics can provide insights into belief formation if there is no or little valuable information on the state of nature. In treating

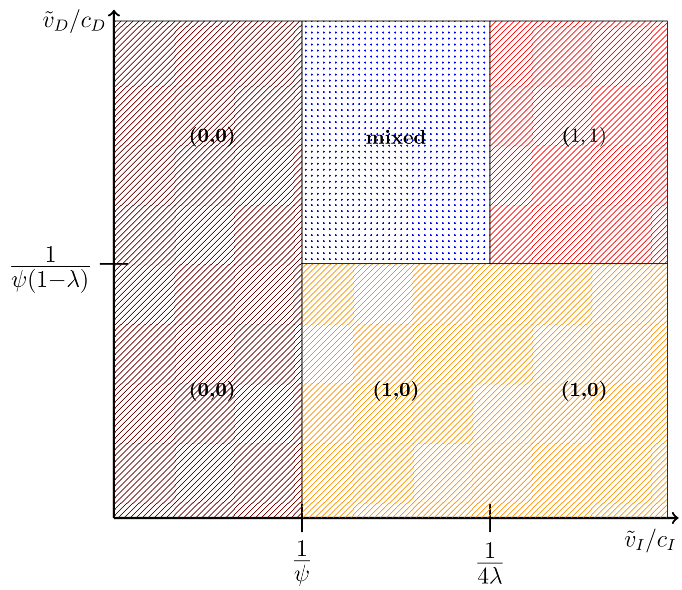

Figure 1: Equilibrium strategy profiles

the distribution of subjective beliefs adopted by the target audience as an equilibrium outcome, our analysis deviates sharply from existing economic analyses of fundamental uncertainty which almost invariably treat subjective (prior) beliefs as primitives.

We now discuss how the different factors assembled by the model determine the subjective beliefs held by the audience members. First consider a scenario with full discounting of after-payoffs ($\delta_D = \delta_I = 0$, implying $\tilde{v}_k = v_k$ for $k = I, D$). In this case, the fundamental has "no voice" in the behavior of any of the players. In particular, the belief entrepreneurs' incentives to campaign are driven by the normalized before-(resolution-of-uncertainty) payoffs, v_k/c_k, which in this scenario are measured on the two axes of Figure 1. The figure can be used to infer how v_I/c_I and v_D/c_D determine the type of equilibrium for given values of λ and ψ—and so influence the set of subjective beliefs adopted by the audience and the resulting demand for the innovation. Things are simple if the innovator's normalized before-payoff is low. Then, there is no campaigning and all $\alpha_i\pi$s, i.e., all subjective beliefs about $\Pr[F = H]$ held by the members of the audience, are identical to zero (equation 6). Therefore, no one opts for the innovative approach (equation 7). If the innovator's normalized before-payoff is higher, the defender's payoff matters, too. First assume that v_D/c_D is low. Then, only the innovator campaigns. If $\zeta = \bar{\zeta}$, the contest produces $\pi = 1/\bar{\alpha}$, inducing all audience members to adopt a subjective belief that prompts the adoption of the innovative approach (equation 7); if $\zeta = 0$, the beliefs are again identical to zero and there is no demand for the innovative approach.

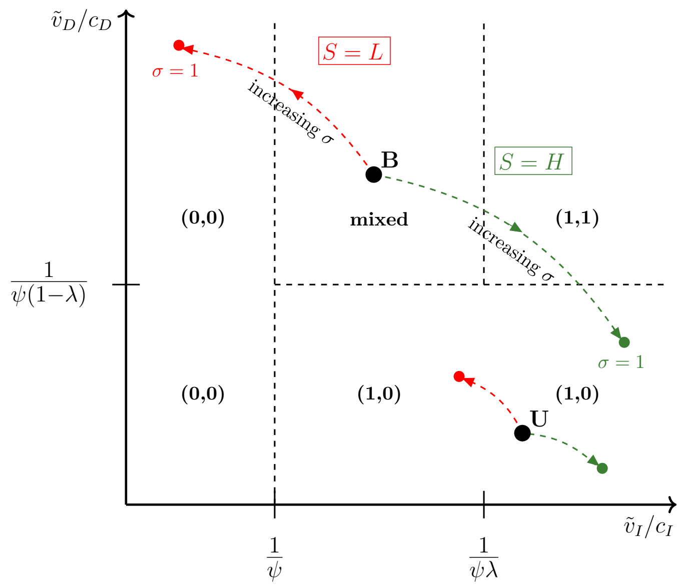

Figure 2: Information and equilibrium strategy profiles

Assume now that v_D/c_D takes a higher value. Then, both belief entrepreneurs have a strictly positive probability of campaigning—with the result that it is less likely that the beliefs about $\Pr[F = H]$ take a high value. Note, however, that in none of the cases discussed, there is any correlation between the fundamental F and the equilibrium beliefs about F.

This may be different in the alternative scenario in which after-payoffs carry some weight: $\delta_k > 0$ for $k = I, D$. Then, information about the fundamental possibly influences the set of beliefs adopted by the audience by altering normalized payoffs, $\tilde{v}_k/c_k$, and so influencing the type of equilibrium. To see how exactly information may find its way into subjective beliefs, consider Figure 2. Assume that prior to the realization of signal $S \in \{H, L\}$ the prevailing $(\tilde{v}_I/c_I, \tilde{v}_D/c_D)$-combination is represented by point B—which means that the normalized payoffs are roughly balanced and sizable. Also assume that the after-payoffs for adverse outcomes are damaging (i.e., w_I^L, w_D^H are "large") and carry substantial weight (i.e., δ_I, δ_D are "large"). Then, when the signal arrives, the belief entrepreneurs update their priors, ϕ_I^0 and ϕ_D^0. Suppose first that $S = H$. In this case, for both entrepreneurs, updating means an increase in the subjective belief about the probability of $F = H$ (i.e., $\phi_I > \phi_I^0$, $\phi_D > \phi_D^0$). So, as compared to the situation prior to the realization of the signal, $\tilde{v}_I$ takes a higher value and $\tilde{v}_D$ takes a lower value. As result, the adjusted $(\tilde{v}_I/c_I, \tilde{v}_D/c_D)$-combination must lie to the south-east of point B—by how much depends on the quality of the signal, as illustrated in the figure. A similar logic implies that $(\tilde{v}_I/c_I, \tilde{v}_D/c_D)$ must lie to the north-west of B if $S = L$. To summarize, in the situation considered (let's call it constellation B), the signal "pushes" the game towards an equilibrium that tends to produce subjective beliefs that are in line with the signal: positive towards the innovation if $S = H$ and negative otherwise.

The lower-right part of Figure 2 visualizes an alternative and less benign situation (called constellation U): in that situation, a combination of a large before-payoff ratio, v_I/v_D, and, e.g., heavy discounting entails that even after a negative realization of the signal ($S = L$) the resulting $(\tilde{v}_I/c_I, \tilde{v}_D/c_D)$-combination is unbalanced in favor of the innovator. As a result, no matter what the signal says, the equilibrium strategy profile is $(1, 0)$. So, unless $\zeta = 0$, everyone will choose the innovative approach (equations 6 and 7).

5.3 Policy Implications

In practice, constellation U may be highly relevant inasmuch as there are good reasons to expect that for many innovations the discounting of after-payoffs is substantial. Conclusive evidence on the success or failure of an innovation often accumulates slowly (see, e.g., Goldfarb and Kirsch 2019) and sometimes the merits of an innovation are not revealed at all before a rare "stress situation" arrives (e.g., as in the case of securitization). Moreover, the tenure of decision makers can be short, a factor that contributes to discounting. So it is not uncommon that before-payoffs, which show little relationship with the fundamentals of an innovation, dominate the incentives to campaign and the formation of subjective beliefs. As a result, the beliefs that the members of a target audience adopt from a belief contest may often not relate in any substantial way to the true merits of the innovation.

Consider a policy maker who does not have access to more information than the target audience but, unlike the audience, does not turn to heuristic belief formation. For such a policy maker, it may be helpful to have an idea about the extent to which one can trust emerging positive subjective beliefs to reflect the fundamentals of an innovation. If the extent is small, the policy maker has to reckon with the possibility that at some point the belief will collide with the facts—and so produce a fallout for which preparations may be required. Our theory offers the policy maker a simple diagnostic framework for assessing such collision potentials. The framework suggests that a policy maker consider information that helps in estimating the size of before-payoffs (i.e., v_I, v_D) and the extent of discounting (i.e., δ_I, δ_D). If the policy maker finds the discount factors δ_k to be small (heavy discounting) and the ratio of before-payoffs v_I/v_D to be large, the situation resembles constellation U in Figure 2. This means that, unless the innovator's narrative is a nonstarter ($\zeta = 0$), the target audience will adopt positive beliefs towards the innovation no matter the signal. As a result, there is no reason to expect that the adoption of positive beliefs by the target audience is informative of the true probability that the state of the world is favorable to the innovation—and fallout preparations appear more urgent. By contrast, constellation B in Figure 2 gives rise to a positive correlation between the subjective beliefs and the probability that the innovation is a success. Worries about a possible fallout may thus appear less pressing.

In essence, our framework suggests that—if objective information on the merits of an innovation is scarce or lacking—a sensible approach is to consider the *incentives* to influence beliefs. How big are the benefits that potentially await the contestants before the resolution of uncertainty? How far away is the resolution of uncertainty? For how long are decision makers typically held to account? It may not be trivial to answer these questions, not least because belief entrepreneurs may try to obscure the facts. But it is far from impossible to estimate these factors with some degree of precision. In any case, obtaining such estimates seems more feasible than identifying the merits of an innovation at an early stage.

6 Belief Formation in the General Setup

We now relax the two simplifying assumptions stated at the beginning of Section 5. The campaigning efforts are now continuous choice variables ($e_k \in \mathbb{R}_{\geq 0}$, $k = I, D$) and the random variable ζ follows a continuous distribution. For tractability, we assume that ζ is uniformly distributed on $\zeta \in [0, u]$, with $u > 0$. For the very same reason, we assume a uniform distribution for the individual components of subjective beliefs, $\alpha_i \in [0, \bar{\alpha}]$.

6.1 Analysis

Individual decisions on the use of the approach follow immediately from equation (3). The equation implies that there exists a threshold $\tilde{\alpha} \equiv \rho/(\bar{\theta}\pi)$ such that $a_i = 1$ if and only if $\alpha_i > \tilde{\alpha}$. Using contest success function (2), and assuming $e_I > 0$, we find

$$\tilde{\alpha}\left(e_I, e_D, \zeta\right) = \frac{\bar{\alpha}\rho}{\bar{\theta}} \left[1 + \frac{e_D^{\eta}}{e_I^{\eta}\,\zeta}\right]. \tag{9}$$

Threshold $\tilde{\alpha}$ determines how the two belief entrepreneurs split the target audience between them. Since the α_is follows a uniform distribution on $[0, \bar{\alpha}]$, it follows that

$$D_I = \max\left\{1 - \tilde{\alpha}/\bar{\alpha},\ 0\right\}. \tag{10}$$

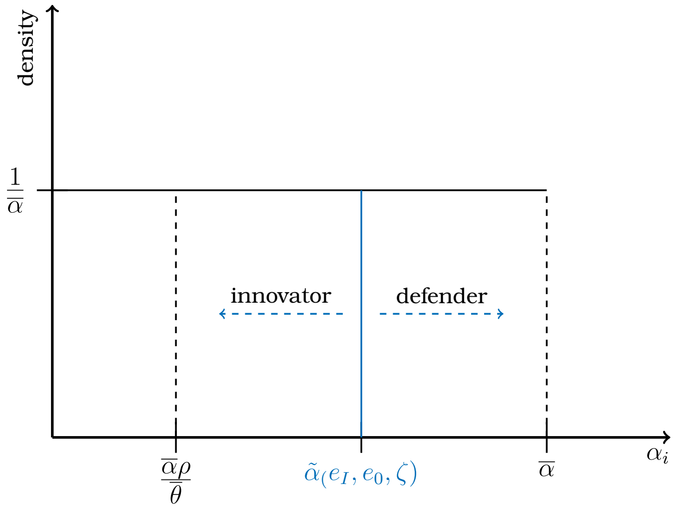

Figure 3: The battle for "market share"

Intuitively speaking, it is the position of $\tilde{\alpha}$ that is contested. In addition to the strategy variables e_I and e_D, $\tilde{\alpha}$ is affected by ζ, a random variable. Finding an optimal response to a given strategy by the opponent is thus an optimization problem under risk. As is illustrated in Figure 3, the innovator would like to push $\tilde{\alpha}$ to the left, while the defender wants to push it to the right. Yet both have only limited control over their "market share" because of random variable ζ. The minimum of $\tilde{\alpha}$ is $\bar{\alpha}\rho/\bar{\theta}$, a level also specified in the figure.

It is noteworthy that $\Pr[\tilde{\alpha} > \bar{\alpha}] = 1$ if e_I takes a small value *relative to* e_D (equation 9). This is because both α_i and ζ have strictly finite upper bounds. Therefore, if e_I is relatively small, the marginal benefit (in terms of a lowering of $\tilde{\alpha}$) associated with raising e_I is 0. When deriving best responses and equilibria, this feature must be taken care of. Moreover, we note that contest success function (2) is not continuous at $(0, 0)$, something that requires additional attention. As a result, objective functions (4) and (5) are not strictly concave, implying that local maxima (as identified by the first- and second-order conditions) do not necessarily mean positive expected payoffs. As the derivation of the equilibrium is tedious, we relegate it to the Appendix. But the characterization of the equilibrium is straightforward:

PROPOSITION 1 *Under individual choice, there exists a unique Nash equilibrium in pure strategies if and only if $\tilde{v}_I/c_I > \nu$ for some critical level $\nu > 0$. In this equilibrium, the ratio of campaigning efforts equals the ratio of normalized payoffs:*

$$\frac{e_I}{e_D} = \frac{\tilde{v}_I/c_I}{\tilde{v}_D/c_D}. \tag{11}$$

Therefore:

$$\pi = \frac{1}{\bar{\alpha}}\left[1+\left(\frac{\tilde{v}_D/c_D}{\tilde{v}_I/c_I}\right)^{\eta}\frac{1}{\zeta}\right]^{-1} \qquad \text{and} \qquad \tilde{\alpha} = \frac{\bar{\alpha}\rho}{\bar{\theta}}\left[1+\left(\frac{\tilde{v}_D/c_D}{\tilde{v}_I/c_I}\right)^{\eta}\frac{1}{\zeta}\right]. \tag{12}$$

Proof. See Appendix B. ■

The expressions in (12) follow immediately from equations (11), (2), and (9). The condition on the normalized payoff stated in the proposition rules out that an interior maximum of V_I is associated with a strictly negative payoff.[14] If that condition is violated, an equilibrium in pure strategies does not exist. Since e_I and e_D are continuous choice variables, mixed equilibria are not straightforward to analyze. In partiular, Nash's existence theorem does not hold. One way of dealing with this constellation is to assume that nature enters as a player and discretizes the action space. The toy model of Section 5 is an extreme example of such a discretization—and its analysis provides insights into why an equilibrium in pure strategies may not exist: if the innovator's normalized payoff is sufficiently small, they want to invest in a belief campaign if and only if the defender does not want to. Under pure strategies, this leads to mutually incompatible incentives because the defender invests in a belief campaign if and only if the innovator invests, too. Note that any use of mixed strategies strengthens the influence of randomness in the process of subjective belief formation.

6.2 Discussion and Policy Implications

The results of the simplified and general setup are very similar. An advantage of the latter is that its predictions regarding equilibrium beliefs are contained in a single, concise equation. The expression for the belief anchor in (12) clarifies how the three factors "information", "payoffs" and "luck" affect equilibrium beliefs. Information enters via the ratio of normalized payoffs, $(\tilde{v}_D/c_D)/(\tilde{v}_I/c_I)$. As in the simplified setup (Figure 2), if $S = H$, the realization of the signal causes the ratio to fall and hence π to adjust in favor of the innovator. Otherwise, if $S = L$, the ratio rises and π moves in favor of the defender. Unlike in the simplified setup (where S matters only if decisive for the equilibrium regime), here the influence of S is continuous. So there is always a positive correlation between equilibrium beliefs and the information conveyed by the signal.[15] By how much the ratio of normalized payoffs adjust in response to the signal is moderated by the payoff structure. For a given signal quality σ, the response is stronger if

[14] We could not find an easily interpretable algebraic expression for ν that is both necessary and sufficient. Conditions that are just sufficient are not particularly insightful.

[15] In formal terms, there is a positive correlation between π and the objective conditional probability of the innovation being a success, $\Pr[F = H|S]$, where $\Pr[F = H|H] = \sigma > 1/2$ and $\Pr[F = H|L] = 1 - \sigma < 1/2$.

the after-payoffs for adverse outcomes are more damaging (i.e., if w_I^L, w_D^H are larger) and carry more weight (i.e., if δ_I, δ_D are larger). But even if the information conveyed by the signal has a strong influence on the ratio of normalized payoffs, (bad) luck with narratives—entering via ζ—can disconnect the beliefs about the innovation from its true merits.

In terms of policy implications, moving from the discrete to the continuous setup leaves matters largely unchanged. The basic suggestion still is that a policy maker who has to deal with an innovation propelled by highly positive subjective beliefs collect information on the payoff structure (before- and after-payoffs, extent of discounting). If the policy maker is confronted with what Subsection 5.2 calls constellation U, heavy discounting renders the signal's influence virtually negligible, with the result that friendly beliefs towards the innovation primarily reflect that the ratio of before-payoffs is unbalanced in favor of the innovator. Those friendly beliefs should therefore be irrelevant for how the policy maker assesses the innovation. By contrast, if the policy maker finds itself in constellation B, the heavy weighting of after-payoffs ensures that information conveyed by the signal has a larger voice—and friendly beliefs should be regarded as a substantive indication in favor of the innovation.

Figure 4 illustrates this logic. The figure shows for each constellation, B and U, 25 simulated realizations of belief anchor π. In both constellations, the signal quality is assumed to be intermediate: $\sigma = 3/4$.[16] This means that the objective conditional probability of the innovation being a success, $\Pr[F = H|S]$, is 3/4 if $S = H$ and 1/4 if otherwise. In constellation B, the realizations of π cluster in the upper part of the $[0, 1]$-range if $S = H$ and in the lower part otherwise. The conditional means of π, marked by the dashes, differ accordingly. While due to ζ the sorting is not perfect, a positive belief towards the innovation is evidence for a high $\Pr[F = H|S]$. The contrast to constellation U is sharp. In the latter constellation, the realizations of π cluster in the upper part of the range no matter what the signal indicates. A favorable belief towards the innovation thus cannot be taken as an indication of its true merits.

7 Conclusion

This paper offers a new perspective on belief formation under fundamental uncertainty. We understand beliefs about novel phenomena as an equilibrium outcome, shaped in a contest between self-interested belief entrepreneurs. The major determinants of equilibrium beliefs are: the belief entrepreneurs' expected benefits ("payoffs"), the degree of fundamental uncertainty ("information"), and randomness relating to the success of narratives ("luck"). A key contri-

[16] Constellation B: $v_I = v_D = 1$ and $\delta = 0.9$. Constellation U: $v_I = 1 > v_D = 1/3$ and $\delta = 0.1$. In both constellations: $w_I^L = w_D^H = 1$, $w_I^H = w_D^L = 0$, $\phi_I^0 = \phi_D^0 = 1/2$, $u = 2$, $\bar{\alpha} = 1$, $\eta = 0.9$, and $\Pr[F = H] = 0.4$.

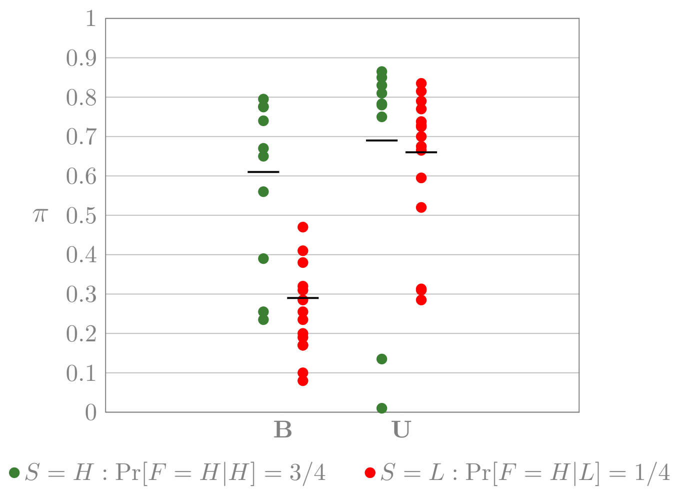

Figure 4: When to trust a favorable belief?

bution of this paper is to show that the microeconomic toolbox can be useful for explaining how beliefs emerge endogenously under fundamental uncertainty.

We see several applications for our framework. Three of them are briefly discussed here. First, our framework may be helpful to regulators that have to assess and potentially regulate innovations that are within their purview. The present analysis identifies some observable factors—such as the innovator's payoff structure—that can be considered in order to gauge the relationship between an emerging subjective belief about an innovation and the innovation's true, but yet unobserved, fundamentals. So far, however, our analysis is stylized. A more elaborate framework useful to regulators confronted with fundamental uncertainty would be dynamic and allow objective information to emerge gradually.

Second, it is natural to ask how endogenous beliefs, as modeled in this paper, affect macroeconomic time series in economies that regularly experience innovation-driven fundamental uncertainty. Clearly, looking into this question requires the specification of how beliefs respond to the gradual resolution of uncertainty. In practice, evidence on innovations often emerges slowly, and so uncertainty resolution takes time. As a result, favorable views about an innovation—even if proven unfounded eventually—may persist for a longer time. But once reality sinks in, the reactions may be severe. In our view, it is interesting to explore to what extent this pattern may help better understand business cycle dynamics, bubble patterns, and the possibilities and limitations of conditionally predicting booms, recessions, and crises.

Third, "ambiguity suppression" is not only observed in connection with innovation-driven fundamental uncertainty but may arise in the context of any complex choice situation. In particular, it is often difficult for voters to "objectively" evaluate the relative costs and benefits of competing policy proposals. As in the case of innovations, this may lead to a demand for heuristic short-cuts—and thus provide political parties (or other players) with an opportunity to act as belief entrepreneurs that offer ready-made evaluation frames. Depending on the details of the contest game, the resulting "narrative campaigns" may (further) polarize the electorate. We find it interesting to explore to what extent this mechanism can shed light on the recent rise in political polarization and populism in the US and elsewhere.

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Appendix for

The Economics of Beliefs under Fundamental Uncertainty

October 31, 2021

A. Collective Choice

In practice, there are many instances in which a decision on an innovation has to be taken collectively (e.g., by a board of directors, by a legislature, or in a popular vote). Appendix A outlines a variant of the model that is based on collective, instead of individual, choice. As this modification leaves the insights of the preceding analysis virtually unchanged, we limit the exposition to the simplified setup presented in Section 5. The analysis of the general setup with collective choice is available from the authors upon request.

Assumptions

Consider the following modification. Under collective choice, each audience member votes for the approach they prefer individually. The audience as a whole then adopts the innovative approach ($a = 1$) if the vote share of the latter (strictly) exceeds a certain threshold, γ; otherwise, the audience chooses the tested approach ($a = 0$). Individual decisions (here: votes) continue to be based on objective function (3), with $a \in \{0, 1\}$ replacing a_i. Specifically, member i votes for the innovative approach if and only if $a = 1$ maximizes $E_i\{U(a)\}$. We denote by $X_I(\pi)$ the share of votes cast in favor of the innovative approach. The functional form of $X_I(\pi)$ is identical to that specified on the right-hand side of equation (7). It follows that the demand for the innovative approach, $D_I(X_I(\pi))$, is 1 if $D_I(X_I(\pi)) > \gamma$ and 0 otherwise. Below, we have to distinguish between two cases: $\lambda \leq \gamma$ and $\lambda > \gamma$.

Defender

Innovator	$e_D = 0$	$e_D = 1$
$e_I = 0$	0 v_D	0 $v_D - c_D$
$e_I = 1$	$\psi\tilde{v}_I - c_I$ $(1-\psi)v_D$	$-c_I$ $v_D - c_D$

Table 2: Game matrix for belief contest with collective choice and $\lambda \leq \gamma$

Defender

Innovator	$e_D = 0$	$e_D = 1$
$e_I = 0$	0 v_D	0 $v_D - c_D$
$e_I = 1$	$\psi\tilde{v}_I - c_I$ $(1-\psi)v_D$	$\psi\tilde{v}_I - c_I$ $(1-\psi)v_D - c_D$

Table 3: Game matrix for belief contest with collective choice and $\lambda > \gamma$

Analysis

First assume that $\lambda \leq \gamma$. In this case, the audience chooses the innovative approach if and only if $\pi = 1/\bar{\alpha}$. The resulting payoffs are shown in Table 2. Compared to Table 1, only the payoffs for $(1,1)$ differ. Since with this strategy profile $X_I(\pi)$ necessarily falls short of γ, the innovator never wins and so has no chance of recovering any cost of campaigning. As a result, profile $(1,1)$ cannot be a Nash equilibrium. Profile $(0,1)$ cannot be an equilibrium either because the defender has an incentive to deviate. The two remaining profiles can be equilibria. Profile $(0,0)$ is an equilibrium if and only if $\tilde{v_I}/c_I \leq 1/\psi$. Profile $(1,0)$ is an equilibrium if and only if $\tilde{v_I}/c_I \geq 1/\psi$ and $v_D/c_D \leq 1/\psi$. If neither of the two latter profiles constitutes an equilibrium, there is an equilibrium in mixed strategies. The probabilities of campaigning are

$$p = \frac{1}{\psi(v_D/c_D)} \qquad \text{and} \qquad q = \frac{\psi - 1/(\tilde{v}_I/c_I)}{\psi}. \tag{A1}$$

As in the case of individual choice, the probability that the innovator campaigns is smaller if the defender is stronger (i.e., has a larger normalized payoff), while the defender's probability of campaigning is larger if the innovator is stronger. Overall, we conclude that the pattern of equilibrium profiles is very similar to that under individual choice. In terms of Figure 1, the most notable difference is that under collective choice the "mixed equilibrium area" extends to the upper-right corner (I and D both campaigning cannot be an equilibrium).

Now consider $\lambda > \gamma$. The payoffs for that case are shown in Table 3. Whenever $e_I = 1$, the innovator wins with probability ψ and the defender wins with the complementary probability. Clearly, for the defender, $e_D = 0$ dominates $e_D = 1$. As a consequence, the unique Nash equilibrium is given by profile $(1, 0)$ if $\tilde{v_I}/c_I > 1/\psi$ and by profile $(0, 0)$ otherwise.

B. Proof of Proposition 1

A profile (e_I, e_D) is an equilibrium if and only if e_k maximizes V_k, given e_{-k}, for both players $k = I, D$—i.e., the profile consists of mutual best responses. In the proof below, we first state the objective functions V_k. We then characterize best responses. We show that any candidate equilibrium needs to fulfill standard first-order conditions, in spite of the fact that the objective functions are neither strictly concave nor everywhere differentiable. However, first-order conditions obviously are not sufficient for a best response. In the final part of the proof, we establish that a sufficiently large normalized payoff makes the first-order conditions also sufficient for a best response and thus an equilibrium.

Belief Entrepreneurs' Objective Functions

We start with the innovator's objective function. In stage 4 of the game, individuals decide on $a_i \in \{0, 1\}$. The resulting demand for the innovative approach, D_I, is given by equation (10). From equation (10), it follows that $D_I = 0$ if $\tilde{\alpha} \geq \bar{\alpha}$, where $\tilde{\alpha}$ is given by equation (9). (For simplicity, we assume that, in case of indifference, a member of the audience chooses $a_i = 0$). Now consider the formation of expectations about D_I in stage 3. From equations (9) and (10), and the fact that the maximum of ζ is u, it follows that

$$E[D_I] > 0 \;\text{ if and only if }\; e_I > \underline{e}_I(e_D), \tag{A2}$$

where

$$\underline{e}_I(e_D) \equiv \left[\frac{\rho}{u(\bar{\theta} - \rho)}\right]^{1/\eta} e_D. \tag{A3}$$

Clearly, $\underline{e}_I(e_D) > 0$ whenever $e_D > 0$. If $0 < e_I \leq \underline{e}_I(e_D)$, $\tilde{\alpha} < \bar{\alpha}$ would require random variable ζ to exceed its maximum of u. Conversely, if $e_I > \underline{e}_I(e_D)$, there is always a strictly positive probability that ζ takes a sufficiently large value such that $\tilde{\alpha} < \bar{\alpha}$ and therefore $D_I > 0$.

Provided that $e_I > \underline{e}_I(e_D)$, straightforward calculations show that

$$D_I > 0 \ \text{ if and only if } \ \zeta > \zeta_0, \tag{A4}$$

where

$$\zeta_0 \equiv \frac{\rho}{\bar{\theta} - \rho} \left(\frac{e_D}{e_I} \right)^{\eta}. \tag{A5}$$

Equation (10), together with (A2) to (A5), implies that—when stage 4 is anticipated—the innovator's objective function (4) can be rewritten as follows:

$$V_I = \begin{cases} \int\limits_{\zeta_0(e_I, e_D)}^{u} \left[1 - \frac{\tilde{\alpha}(e_I, e_D; \zeta)}{\bar{\alpha}} \right] \frac{1}{u} \, \mathrm{d}\zeta \, \tilde{v}_I - c_I e_I & \text{if } \ e_I > \underline{e}_I(e_D), \ e_D > 0 \\ \left(1 - \frac{\rho}{\bar{\theta}}\right) \tilde{v}_I - c_I e_I, & \text{if } \ e_I > 0, \ e_D = 0 \\ -c_I e_I & \text{if } e_I \leq \underline{e}_I(e_D), \ e_D > 0 \\ 0 & \text{if } e_I = 0, \ e_D = 0 \end{cases}. \tag{A6}$$

In equation (A6), the integral on the first line gives $E\left[D_I\right]$ for the specified (e_I, e_D)-combination. Because of (A4), ζ_0 marks the lower integration limit (which depends on the campaigning efforts, too). If $e_I > 0$ and $e_D = 0$, demand simplfies to $D_I = 1 - \rho/\bar{\theta}$, as shown on the second line. Because the minimum value of $\tilde{\alpha}$ is $\bar{\alpha}\rho/\bar{\theta} > 0$, the innovator does never server the entire target audience. The third line in equation (A6) refers to the situation in which e_I is not sufficiently large to generate positive demand for the innovative approach. To understand the final line, note that for this case equation (2) specifies $\pi = 0$ (and thus $\tilde{\alpha} \to \infty$).

Turning to the defender's objective function, we note that in stage 4 she serves a fraction $D_D = \min\left\{\tilde{\alpha}/\bar{\alpha}, \ 1\right\}$ of the target audience. Because $\tilde{\alpha} \geq \bar{\alpha}\rho/\bar{\theta}$, there is always some demand for the tested approach. Moreover, $D_D = 1$ whenever $\zeta < \zeta_0$. Furthermore, if e_D is sufficiently large, the defender serves the entire audience even if $\zeta = u$:

$$\text{If } \ e_D \geq \bar{e}_D(e_I), \ \ \text{then } \ D_D = 1, \tag{A7}$$

where

$$\bar{e}_D(e_I) \equiv \left(\frac{u(\bar{\theta} - \rho)}{\rho} \right)^{\frac{1}{\eta}} e_I. \tag{A8}$$

So, if $e_D \geq \bar{e}_D$, an increase in e_D is useless. All this implies that—when stage 4 is anticipated—

the defender's objective function (5) can be rewritten as follows:

$$V_D = \begin{cases} \dfrac{\zeta_0(e_I,e_D)}{u}\tilde{v}_D + \displaystyle\int\limits_{\zeta_0(e_I,e_D)}^{u} \dfrac{\tilde{\alpha}(e_I,e_D;\zeta)}{\bar{\alpha}}\dfrac{1}{u}d\zeta\ \tilde{v}_D - c_D e_D & \text{if } \ e_D < \bar{e}_D(e_I),\ e_I > 0 \\ \tilde{v}_D - c_D e_D & \text{otherwise} \end{cases}. \tag{A9}$$

Note that for $e_D = 0$, we have $\zeta_0 = u$.

Best Responses

We start again with the innovator. Taking the partial derivative of V_I with respect to e_I yields:

$$\frac{\partial V_I}{\partial e_I} = \begin{cases} \displaystyle\int\limits_{\zeta_0}^{u} \frac{\rho\eta}{\bar{\theta}u}\frac{e_D^{\eta}}{e_I^{1+\eta}}\frac{1}{\zeta}d\zeta\ \tilde{v}_I - c_I & \text{if } e_I \geq \underline{e}_I(e_D),\ e_D > 0 \\ -c_I & \text{if } 0 \leq e_I < \underline{e}_I(e_D),\ e_D > 0;\ \text{or if } e_I > 0,\ e_D = 0 \\ \infty & \text{if } e_I = 0,\ e_D = 0 \end{cases}. \tag{A10}$$

To understand the first line of equation (A10), note that ζ_0—defined in equation (A5)—is a function of e_I and marks the lower limit of the integral on the first line of equation (A6). In the partial derivative, this gives rise to a term $[1 - \tilde{\alpha}(e_I, e_D, \zeta_0)/\bar{\alpha}]\, u^{-1}\, (\partial\zeta_0/\partial e_I)\, \tilde{v}_I$. However, because $\tilde{\alpha}(e_I, e_D, \zeta_0) = \bar{\alpha}$ by definition of ζ_0 (see equations (9) and (A5)), this term vanishes. The third line of equation (A10) uses somewhat informal notation. It refers to the right-sided derivative (since $e_I \geq 0$) and expresses that V_I makes a discrete jump from 0 to a strictly positive value if e_I increases marginally from 0.

Consider the case $e_D = 0$. Then, the third line of the right-hand side of equation (A10) implies that $e_I = 0$ is not a best response, implying that the profile $(0, 0)$ cannot be an equilibrium. Furthermore, if $e_D = 0$, the second line implies that no $e_I > 0$ is a best response: the innovator could marginally decrease e_I to a level that is still strictly positive, with no reduction in market share. This follows from the discontinuity of the contest success function at zero. As a result, no profile with $e_D = 0$ can be an equilibrium.

For the case $e_D > 0$, equation (A10) implies that $0 < e_I < \underline{e}_I$ cannot be a best response because $\partial V_I/\partial e_I < 0$ and lowering e_I is feasible. Solving the integral on the first line yields

$$\frac{\partial V_I}{\partial e_I} = \frac{\rho\eta}{\bar{\theta}u}\frac{e_D^{\eta}}{e_I^{1+\eta}}\left[\ln\left(\frac{u(\bar{\theta}-\rho)}{\rho}\right) - \eta\ln\left(\frac{e_D}{e_I}\right)\right]\tilde{v}_I - c_I \quad \text{if} \quad e_I \geq \underline{e}_I,\ e_D > 0. \tag{A11}$$

The second partial derivative for this constellation is given by

$$\frac{\partial^2 V_I}{\partial e_I^2} = \frac{\rho\eta}{\bar{\theta}u}\frac{e_D^\eta}{e_I^{2+\eta}}\left[\eta - (1+\eta)\ln\left(\frac{u(\bar{\theta}-\rho)}{\rho}\frac{e_I^\eta}{e_D^\eta}\right)\right]\tilde{v}_I \quad \text{if} \quad e_I > \underline{e}_I,\ e_D > 0. \tag{A12}$$

Note that for $e_I = \underline{e}_I$ we have $u(\bar{\theta}-\rho)\rho^{-1}(e_I/e_D)^\eta = 1$. As a result, $\lim_{e_I \searrow \underline{e}_I} \partial V_I/\partial e_I = -c_I < 0$ and $\lim_{e_I \searrow \underline{e}_I} \partial^2 V_I/\partial e_I^2 > 0$. Moreover, the term in square brackets in equation (A12) is a strictly decreasing function of e_I and eventually turns negative. Finally, $\lim_{e_I \to \infty} \partial V_I/\partial e_I = -c_I$ since a power function grows sufficiently faster towards infinity than its logarithm. Thus, as e_I rises from $\underline{e}_I$ towards ∞, $\partial V_I/\partial e_I$ (strictly) increases from $-c_I$, reaches a maximum, and then (strictly) falls towards the initial level. Depending on $\tilde{v}_I/c_I$, as well as on other parameters, $\partial V_I/\partial e_I = 0$ has either zero, one, or two solutions (the situation with only one solution is a limit case). If there are two solutions, $\partial V_I/\partial e_I$ crosses zero from below at the level of the smaller solution and from above at the level of the larger. So the second-order condition holds for the larger solution only. To summarize the results for the innovator's best response:

LEMMA 1 *(i) If $e_D = 0$, no best response exists; as a result, no profile (e_I, e_D) with $e_D = 0$ can be an equilibrium. (ii) If $e_D > 0$, the innovator's best response is either $e_I = 0$ or given by the larger of the two solutions to $\partial V_I/\partial e_I = 0$, where $\partial V_I/\partial e_I$ is given by equation (A11).*

We now turn to the defender. Taking the partial derivative of V_D with respect to e_D yields:

$$\frac{\partial V_D}{\partial e_D} = \begin{cases} \displaystyle\int_{\zeta_0}^{u} \frac{\rho\eta}{\bar{\theta}u}\frac{1}{e_I^\eta e_D^{1-\eta}}\frac{1}{\zeta}d\zeta\ \tilde{v}_D - c_D & \text{if} \ \ e_D < \bar{e}_D(e_I),\ e_I > 0 \\ -c_D & \text{otherwise} \end{cases} \tag{A13}$$

Again note that e_D appears in the lower limit of the integral on the first line of equation (A9). The corresponding derivative term cancels with the one obtained for the first term of the same line of equation (A9).

Suppose that $e_I = 0$ (such that $\bar{e}_D(e_I) = 0$). Then, the second line of equation (A13) implies that no $e_D > 0$ can be a best response. Therefore, the best response is $e_D = 0$. Consider now $e_I > 0$. The second line of equation (A13) implies that $e_D > \bar{e}_D$ is never a best response in this case. Solving the integral in the first line yields

$$\frac{\partial V_D}{\partial e_D} = \frac{\rho\eta}{\bar{\theta}u}\frac{1}{e_I^\eta e_D^{1-\eta}}\left[\ln\frac{u(\bar{\theta}-\rho)}{\rho} - \eta\ln\left(\frac{e_D}{e_I}\right)\right]\ \tilde{v}_D - c_D \quad \text{if} \quad e_D < \bar{e}_D,\ e_I > 0. \tag{A14}$$

Clearly, $\lim_{e_D \to 0} \partial V_D/\partial e_D = \infty$ in this constellation (recall that $0 < \eta < 1$). It follows that

$e_D = 0$ is never a best response to any $e_I > 0$. As e_D approaches $\bar{e}_D$ from below, the expression in square brackets in equation (A14) approaches 0. Hence, we have $\lim_{e_D \nearrow \bar{e}_D} \partial V_D/\partial e_D = -c_D$ for the respective constellation. Moreover, it is straightforward to show that, for $e_D < \bar{e}_D$ and $e_I > 0$, we have $\partial^2 V_D/\partial e_D^2 < 0$. To summarize the results for the defender's best response:

LEMMA 2 *(i) If $e_I = 0$, the defender's best response is $e_D = 0$. (ii) If $e_I > 0$, the first-oder condition $\partial V_D/\partial e_D = 0$, where $\partial V_D/\partial e_D$ is given by equation (A14), has a unique solution, $e_D^* \in (0, \bar{e}_D)$. This solution is the unique best response and the second-order optimality condition for an interior optimum holds. Moreover, $e_D = 0$ is never a best response to any $e_I > 0$.*

Equilibrium

Lemmas 1(i) and 2(i) imply that no equilibrium involves $e_I = 0$ or $e_D = 0$. Moreover, the lemmas' second parts imply that, if an equilibrium exists, the corresponding best responses are given by unique interior solutions characterized by standard first-order conditions:

LEMMA 3 *If an equilibrium exists, it is unique and determined by the first-oder conditions $\partial V_I/\partial e_I = 0$ and $\partial V_D/\partial e_D = 0$, where the two derivatives are given by equations (A11) and (A14), respectively.*

Setting the expressions in equations (A11) and (A14) to zero, we obtain equation (11), the key equation of Proposition 1. The expressions in (12), which are also part of the proposition, follow immediately from equations (11), (2), and (9).

However, since the objective functions (A6) and (A9) are neither strictly concave nor everywhere differentiable, the first- and second-order conditions may not be sufficient for a best response (there may be corner solutions). To complete the proof, we need to verify three more properties of the candidate equilibrium choices as given by the first-order conditions: (i) $e_I > \underline{e}_I > 0$ and $0 < e_D < \bar{e}_D$ (as a consistency check); (ii) the second-order conditions (again as a consistency check); (iii) both players are better off by choosing $e_k > 0$ rather than $e_k = 0$, $k = I, D$. In other words, appropriate versions of "participation constraints" for the belief contest must hold. Specifically, we must show that these conditions hold if and only if $\tilde{v}_I/c_I$ is sufficiently large.

By setting the expressions in equations (A11) and (A14) to zero, and then solving the resulting system of equations for e_I and e_D, we obtain

$$e_I = \frac{\rho\eta\, \tilde{v}_I/c_I}{\bar{\theta} u} \left(\frac{\tilde{v}_D/c_D}{\tilde{v}_I/c_I} \right)^{\eta} \left[\ln\left(\frac{u(\bar{\theta} - \rho)}{\rho} \right) - \eta \ln\left(\frac{\tilde{v}_D/c_D}{\tilde{v}_I/c_I} \right) \right] \tag{A15}$$

and

$$e_D = \frac{\rho\eta\,\tilde{v}_D/c_D}{\bar{\theta}u}\left(\frac{\tilde{v}_D/c_D}{\tilde{v}_I/c_I}\right)^{\eta}\left[\ln\left(\frac{u(\bar{\theta}-\rho)}{\rho}\right)-\eta\ln\left(\frac{\tilde{v}_D/c_D}{\tilde{v}_I/c_I}\right)\right]. \tag{A16}$$

Clearly, $e_I > 0$, $e_D > 0$ if and only if in equations (A15) and (A16) the expression in square brackets is strictly positive. This is the case if and only if

$$\frac{\tilde{v}_I/c_I}{\tilde{v}_D/c_D} > \left(\frac{\rho}{u(\bar{\theta}-\rho)}\right)^{\frac{1}{\eta}}. \tag{A17}$$

Moreover, using equations (A3), (A8), and (11), it follows that $e_I > \underline{e}_I$ and $e_D < \bar{e}_D$ if and only if equation (A17) holds. Since a sufficiently high value of $\tilde{v}_I/c_I$ implies condition (A17), it also implies $e_I > \underline{e}_I > 0$ and $0 < e_D < \bar{e}_D$. So we have checked condition (i) from above.

Next consider the second-order conditions. For e_D, the second-order condition obviously holds because of Lemma 2(ii). Regarding e_I, it is clear from equation (A12) that the sign of $\partial^2 V_I/\partial e_I^2$ exclusively depends on the sign of the term in square brackets. By using equation (11) in equation (A12), and then rearranging terms, we obtain that the term in square brackets is strictly negative if and only if

$$\frac{\tilde{v}_I/c_I}{\tilde{v}_D/c_D} > \left(\frac{\rho}{u(\bar{\theta}-\rho)}\right)^{\frac{1}{\eta}} e^{\frac{1}{1+\eta}}, \tag{A18}$$

where e denotes Euler's number. Since $e^{1/(1+\eta)} > e^{1/2} > 1$, condition (A18) is stricter than condition (A17), implying that we can ignore the latter. This verifies condition (ii) from above.

Finally, we need to check whether the belief entrepreneurs' "participation constraints" hold, i.e., that they are not better off by choosing a corner solution $e_k = 0$, $k = I, D$. Using equations (A5), (9) (11), and (A15) in the first line of equation (A6) results in

$$V_I = \frac{\tilde{v}_I}{\bar{\theta}u}\left\{(\bar{\theta}-\rho)u - \rho\left(\frac{\tilde{v}_D/c_D}{\tilde{v}_I/c_I}\right)^{\eta}\left[1+(1+\eta)\left(\ln\frac{u(\bar{\theta}-\rho)}{\rho}-\eta\ln\left(\frac{\tilde{v}_D/c_D}{\tilde{v}_I/c_I}\right)\right)\right]\right\}. \tag{A19}$$

With $\tilde{v}_I/c_I$ increasing towards infinity, V_I approaches $\tilde{v}_I/\left(\bar{\theta}u\right)(\bar{\theta}-\rho)u > 0$. This follows from the fact that a power function grows sufficiently faster towards infinity than its logarithm. Therefore, $V_I > 0$ holds if $\tilde{v}_I/c_I$ is sufficiently large.

Note that V_D is always positive since the defender serves at least a fraction $\bar{\alpha}\rho/\bar{\theta}$ of the target audience (even without contest participation). Thus, the defender's "participation constraint" concerning the contest is $V_D > \rho/\bar{\theta}\tilde{v}_D$. However, since $\lim_{e_D\to 0}\partial V_D/\partial e_D = \infty$ for any $e_I > 0$, this participation constraint is implied by the first-order condition. Even so, for completeness, we state the equilibrium value of V_D:

$$V_D = \frac{\rho \tilde{v}_D/c_D}{u}\left[\frac{1}{\bar{\theta}-\rho}\left(\frac{\tilde{v}_D/c_D}{\tilde{v}_I/c_I}\right)^{\eta} + \frac{1}{\bar{\theta}}\left(u - \frac{\rho}{\bar{\theta}-\rho}\left(\frac{\tilde{v}_D/c_D}{\tilde{v}_I/c_I}\right)^{\eta}\right)\right.$$
$$\left. + \frac{1-\eta}{\bar{\theta}}\left(\frac{\tilde{v}_D/c_D}{\tilde{v}_I/c_I}\right)^{\eta}\left(\ln\frac{u(\bar{\theta}-\rho)}{\rho} - \eta \ln\left(\frac{\tilde{v}_D/c_D}{\tilde{v}_I/c_I}\right)\right)\right]. \quad \text{(A20)}$$

We conclude that a sufficiently large value of $\tilde{v}_I/c_I$ also implies that condition (iii) holds.

In sum, if $\tilde{v}_I/c_I$ is sufficiently large, the profile specified by equations (A15) and (A16)—which is derived from first-order conditions—indeed consists of mutual best responses and hence constitutes an equilibrium in pure strategies. In this equilibrium, (11) and (12) hold.

To conclude the proof, we need to consider the converse: if $\tilde{v}_I/c_I$ is not sufficiently large, no equilibrium in pure strategies exists. Denote by ν' a critical level for $\tilde{v}_I/c_I$ such that, if $\tilde{v}_I/c_I > \nu'$, conditions (i) – (iii) as stated above hold. Define ν as the smallest such critical level. Then, whenever $\tilde{v}_I/c_I < \nu$, at least one of the conditions (i) – (iii) is violated and hence no (pure-strategy) equilibrium exists.